Simulation of Laminar Natural Convection of Square Enclosure with Partially Heated Adjacent Vertical and Horizontal Walls

Adil Abbas.Mohammed Mechanical Engineering Departement College of Engineering Almustansiriya University Baghdad Iraq E. <u>mail:adekef@yahoo.com</u>

Abstract. The present study represents investigation of the laminar natural convection phenomena in an enclosed spaces. Realization of this subject has been done through different Raleigh numbers and aspect ratios, which have to make better understanding and configuration temperature, stream lines and vorticity fields in enclosures. The real physical model of the enclosure, which represents two-dimensional square object with partially heated adjacent vertical and horizontal walls and adiabatic remaining horizontal walls. Physical model represents base for mathematical model which defines valid parameters for temperature flow regime. The two dimensional differential conservation equations of mass, momentum and energy are solved by a finite difference method. Air was chosen as a working fluid (Pr=0.7), for Rayleigh number varying from 10^3 to 10^7 . The changes in temperature and flow fields (stream functions) with increase in Rayleigh number are investigated for different (Ar). The isothermal cold sections adjacent to the vertical heat source assist the development of secondary circulation cells, that depend upon both Rayleigh number and (Ar). With partial heating and cooling sections on one wall, the flow is characterized by boundary layers lining these sections with separate circulation cells. It is found that Nusselt number is an increasing function of Rayleigh number. For vertical heat source Nu increases with decreasing (Ar), for horizontal heat source Nu increases with increasing (Ar). Results confirmed by previous experiments. Key word: Natural Convection, Laminar, Enclosure.

المستخلص

Nomenclatures.

- Ar Aspect Ratio = H/L
- E Ratio of the heat source size to the total height (ℓ/H)
- G_r Gratshof number
- H Hight, m
- k Thermal conductivity, W/m k
- L Enclosure width, m
- ℓ Heat source length, m
- Nu Nusselt number
- Nuav Average Nusselt number
- p Pressure, N/m²
- P_r Prandtil number
- Ra Rayleigh number
- S Distance of the heat source center, m.
- T_H High Temperature , °C
- T_C Low Temperature , $^{\circ}C$
- u velocity in x-direction, m/s^2
- v velocity in y-direction , m/s^2
- ρ density, kg/m³
- θ Dimensionless temperature
- β Coefficient of volumetric thermal expansion , K⁻¹
- Ψ Dimensionless stream function
- Ω Dimensionless Vorticity function
- ε Dimensionless heat source length = $\ell/W = \ell/L$

Subscripts

- i Nodal point in x-direction
- j Nodal point in y-direction

1. Introduction.

Heat transfer by natural convection, the fluid flows naturally as it is driven by the effect of buoyancy driven motion caused by the body force field. The natural convection flow phenomena inside an enclosed space, is an interesting example of very complex fluid systems that may yield to analytical, empirical and numerical solutions. The enclosure is defined as the confined space bounded by walls of any shape and filled with fluids. Natural convection in enclosures is always created by the complex interaction between the fluid and the heat with all the walls. Internal interactions will cause a diversity of flows that can appear inside the enclosures. In many engineering applications and naturally occurring processes, natural convection in enclosures provides a useful description of the confined fluids in many practical situations ^[2]. Heat transfer by buoyancy driven flow is of importance for a large number of engineering applications ventilation of buildings, thermal performance of heat storage tanks ^[3]. Natural convection in differentially heated enclosures provides means of heat transfer without the need for fans or pumps ^[4].

Free convection in enclosures are found in double-glazed windows, solar collectors, building walls, concentric cryogenic tubes and electronic packages. Natural convection cooling of components attached to printed circuit boards which are placed vertically in an enclosure is currently of great interest to the microelectronics industry. As a consequence of

packing a very large number of components into one very small chip, the attendant volumetric heat generation rate has risen to extremely high levels. A fluid in an enclosed space experiences free convection if the walls of the enclosure are not at a uniform temperature. A buoyancy force causes the fluid to circulate in the enclosure transferring heat from the hot side to the cold side.

If buoyancy forces are not large enough to overcome viscous forces, circulation will not occur and heat transfer across the enclosure will essentially be by conduction. Natural convection in a closed square cavity has occupied the center stage in many fundamental heat transfer analysis which is of prime importance in certain technological applications. In fact, buoyancy-driven convection in a sealed cavity with differentially heated isothermal walls is a prototype of many industrial application such as energy efficient design of buildings and rooms, operation and safety of nuclear reactors and convective heat transfer associated with boilers. For confined natural convection, in contrast, boundary layers form near the walls but the region external to them is enclosed by the boundary layers and forms a core region.

Since the core is partially or fully encircled by the boundary layers, the core flow is not readily determined from the boundary conditions but depend on the boundary layer, which, in turn, is influenced by the core. The interactions between the boundary layer and core constitute a major complexity in the problem. In fact, the situation is even more intricate because it often appears that more than one global core flow is possible and flow subregions, such as, cells and layers, may be embedded in the core ^[5]. Various configurations for enclosures have been investigated using different solution techniques and for different side heating conditions, such as asymmetrically heated opposite vertical or inclined sides and discrete wall heating. The scope of these studies covered a wide range of parameters influencing the heat transfer process such as the number of discrete heat sources and their locations, the cavity width, effect of Prandtl number and the effect of aspect ratio ^[6]. Several attempts have been made to acquire a basic understanding of natural convection flows and heat transfer characteristics in an enclosure.

However, in most of these studies, one vertical wall of the enclosure is cooled and another one heated while the remaining top and bottom walls are well insulated. Fairosidi Bin Idrus,2008 ^[1], focused on the numerical investigation of a steady laminar natural convection flow of air with Prandtl number, Pr = 0.71 in a two-dimensional enclosure. For the analysis, two cases with different boundary conditions are investigated; two vertical walls, which are isothermal and two verticals walls, which are non-isothermal. For both conditions, adiabatic horizontal walls and Rayleigh number, Ra in low range of $10^3 - 10^5$, are taken into consideration.

The results are presented in the tabular form of the average Nusselt number, Nu and the graphical form of the isotherms and velocity vector contours. Orhan Aydin & Wen-Jei,2000 ^[2], studied natural convection of air in a two dimensional rectangular enclosure with localized heating from below and symmetrical cooling from the sides has been numerically investigated. Localized heating is simulated by a central located heat source on the bottom wall. The average Nusselt number was shown to increase with an increase the (Ra).

Abdulhaiy & Jalal,2000^[3] studied natural convection heat transfer in a square air-filled enclosure with one discrete flush heater. The enclosure is vertical with isothermal heating strip on one wall, the remainder of the wall and the opposing one are isothermally maintained at a lower temperature. The top and bottom sides are adiabatic.

The changes in temperature and flow fields (stream functions) with increase in Rayleigh number are investigated for different heater locations. The isothermal cold sections adjacent to the heater assist the development of secondary circulation cells, that depend upon both Rayleigh number and the position. With discrete heating and cooling sections on one wall,the flow is characterized by boundary layers lining these sections with separate circulation cells. The variation of the local Nusselt number is influenced by this flow pattern and the average Nusselt number is higher than that of a discrete heating strip mounted on an adiabatic wall. The optimum location over the range of Rayleigh number is for the heater mounted at the center of the wall, S/H = 0.5 a result confirmed by previous experiments.

Al-Bahi *et al*,2002 ^[4], Laminar natural convection heat transfer in an air filled vertical square cavity differentially heated with a single isoflux discrete heat source on one wall with top and bottom adiabatic surfaces is numerically studied. The coupled unsteady two dimensional conservation equations are solved by employing a forward time central space implicit finite difference scheme. Numerical results reveal that the local Nusselt number decreases along the length of the heater at constant value of the Rayleigh number (*Ra*) with slight enhancement near the trailing edge. The discrete heat source for the maximum heat dissipation rate is Rayleigh number dependent, which is in agreement with previous experimental and numerical results. A correlation for this location is obtained in addition to a relation for the dependence of the average Nusselt number on the Rayleigh number and the location parameter (*S/L*).

Average Nusselt number variation for S/L = 0.25 to 0.75 and $Ra = 10^3$ to 10^6 is presented and compared to the available correlations for full contact and discretely heated enclosures studied numerically. Refai & Yavanovich,1992 ^[6] studied natural convection from discrete heat source in vertical square enclosure, a discrete heat source is located in the centre of one vertical side representing a high power integrated circuit (IC). The conservation equations are solved using the primitive variables: velocity, pressure, and temperature. Computations are carried out for Pr = 0.72, Ar= 1 and 0 <Ra< 10⁶ (Rayleigh number is based on the length of the heat source S divided by the aspect ratio A).The ratio E of the heat source size to the total height lies in the range 0.25 < E < 1.0.

Verification of numerical results are obtained at Ra =0 (conduction limit) with an analytical conduction solution, and the dependence of Nu and total resistance on Ra, E, and boundary conditions are studied. Relationships between Nu and Ra based on different scale lengths are examined. In addition, a relationship between Nu and Ra, based on the characteristic length for an enclosure with a discrete heat source. Nor Azwadi & Rosdzimin ,2008 ^[11] studied two-dimensional natural convection in a square cavity, lattice Boltzmann method, a numerical tool based on the particle distribution function is used to simulate thermal fluid flow problem. A well-known finite difference technique with third order

accuracy in space is combined with the double population thermal lattice Boltzmann model. The basic idea is to solve the velocity and the temperature field using two different distribution functions.

The simulation of localized heated bottom and symmetrical cooling from the sides were carried out in order to validate this approach. The combination of finite difference with double population thermal lattice Boltzmann model is found to be an efficient and stable numerical approach for low and moderate Rayleigh number calculations. Santhosh, 2009 ^[13] study natural convection in differentially heated enclosure, CFD code of natural convection with variable properties and slip condition are presented in this work. The 2D, laminar simulations are obtained by solving the governing equations using a Fluent 6.2.16. It is considered that the temperature variations are not so high and the Boussinesq approximation is applied. The latter leads to the simplification of the system of equations. The computed results for Nusselt number, velocity and temperature profiles and heat transfer rate are directly compared with those proposed in the bibliography letting therefore the validation of the employed numerical procedure.

The aim of this study is to use a numerical finite difference technique to obtain solutions of a two-dimensional model of a square enclosure with laminar natural convection heat transfer from discrete heat sources, a discrete heat source is located in the adjacent vertical and horizontal walls.

2. Mathematical Formulation

The present study is done for a case with a configuration as shown in Fig(1), which is two dimensional square enclosure with a side of length (L),high (H) and adiabatic top and bottom wall. The left vertical and below sides have a flush discrete heater at a constant temperature T_H . The rest of the left vertical wall as well as the right side, are kept at a lower temperature T_C . The hot isothermal strips are located in positions adjacent to each other consist corner heat source, the length of the strip (ℓ), the high of the center of the heat source (S).

The fluid inside the enclosure is assumed incompressible, Newtonian with density variation only pertinent to temperature changes.



Fig (1). Type of enclosure considered.

The governing mass, momentum (x and y) directions and energy conservation equations for steady state buoyancy driven fluid flow are:-

2.1 Mathematical Model

2.1.1 Continuity Equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

2.1.2 Momentum Equation

The momentum transport equation includes the buoyancy force generated as a result of difference in density of the fluid caused by the temperature difference. The buoyancy term is computed based on the Boussinesq approximation ^[7]

2.1.2.1 X-Momentum

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + v\frac{\partial^2 v}{\partial x^2} + v\frac{\partial^2 v}{\partial y^2} + g\beta(T - T_{\infty}) \quad \dots \dots \dots (3)$$

2.1.3 Energy Equation

2.1.4 Vorticity Equation

The following dimensionless variables will be used

$$\Theta = \frac{T - T_c}{T_H - T_c} \qquad (7)$$

Eliminating the pressure gradient terms in Eqs. 2 and 3 by cross partial differentiation and introducing the vorticity function: The final shape of the equations:-

$$\frac{\partial \Psi}{\partial Y} \frac{\partial \Omega}{\partial X} + \frac{\partial \Psi}{\partial X} \frac{\partial \Omega}{\partial Y} = P_r \left(\frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) - \frac{\beta g (T_H - T_C) L^3 P_r^2}{v^2} \frac{\partial \theta}{\partial X} \quad \dots \tag{9}$$

$$R_{a} = \frac{\beta g(T_{H} - T_{C})L}{\nu^{2}} P_{r} = G_{r} P_{r} \qquad (10)$$

$$\frac{\partial^{2} \Omega}{\partial X^{2}} + \frac{\partial^{2} \Omega}{\partial Y^{2}} = \frac{1}{P_{r}} \left(\frac{\partial \Psi}{\partial Y} \frac{\partial \Omega}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \Omega}{\partial Y} \right) + R_{a} \frac{\partial \theta}{\partial X} \qquad (11)$$

$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\mathbf{\Omega}$		(12)
$\frac{\partial^2 \theta}{\partial \theta} \perp \frac{\partial^2 \theta}{\partial \theta} = \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial \theta} = \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial \theta}$		(13)
$\frac{\partial X^2}{\partial X^2} + \frac{\partial Y^2}{\partial Y^2} - \frac{\partial Y}{\partial Y} \frac{\partial X}{\partial X} - \frac{\partial X}{\partial X} \frac{\partial Y}{\partial Y}$	••••••	(13)

3. Numerical Solution

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3.1 Nusselt calculation

Fourier's law gives at nodal points on hot wall

$$q_w = \left[-k\frac{\partial T}{\partial x}\right]_{x=0} \tag{17}$$

Which becomes at dimensionless variables ^[14]:

$$Nu_{i,j} = \frac{q_W L}{k(T_H - T_C)} = \left[-\frac{\partial \theta}{\partial X}\right]_{i,j}$$
(18)

$$Nu_{av} = \frac{1}{H/w} \int_0^H Nu_{(0,j)} dx$$
 (19)

3.2 Boundry Conditions

4. Numerical Results and Discussion.

The study considers buoyancy driven motion of air created by a strip heater. The dimensionless length of the heater, ℓ / H , ℓ / L , are constant with the relative location S/H, S/L at 0.17, this position refer to placing the heated sections at the bottom corner.

4.1 Flow and Temperature Fields.

The development of the isothermal lines, stream and vorticity functions as Rayleigh number varies from 10^3 to 10^7 , with the adiabatic upper and partially lower wall. At Rayleigh number, Ra= 10^3 heat dissipated from the discrete heater develops a fluid layer, the flow forms a single circulation cell with its centre coinciding with that of the enclosure. A small eddy circulation cell appears in the upper left corner, the colder stream had no choice but discharge horizontally from air of the enclosure $^{[10]}$, that moves upward at low velocity as shown in Fig(2,a₁,b₁,c₁), an increase in Rayleigh number to Ra = 10^4 Fig(2,a₂,b₂,c₂) causes distortion of the isotherms indicated largely spaced isotherms . For Ra= 10^5 (2,a₃,b₃,c₃), Ra= 10^6 Fig(3,a₁,b₁,c₁), Ra= 10^7 (3,a₂,b₂,c₂), there are a separation in stream lines due to high circulations. where the heat propagates more towards the cold wall opposite to the discrete heater where the main cell is distorted and prolonged to an elliptic shape.

The range of aspect ratio in this study starts from 0.5 to 3, Fig(4), shows the effect of aspect ratio (Ar) on developing of isothermal lines at Ra=10³, air in the cavity rises along the hot wall and descends along the cold wall. The isothermal lines shows a tilt to the right side because the double effect of heating from left and bottom sides. the flow or the isotherms contours were occupied the whole cavity more uniformly .Fig(5), shows the effects of high Rayleigh number (Ra=10⁶) and changing in (Ar).

At high (Ra) corresponds to a thinner boundary layer, and high circulation the effect is clear in the center core. Increasing in (Ar) shows a larger center core, but in decreasing (Ar), causes high isothermal lines circulation which is clear in the core center ^[8].



Fig(2). Distributions of temperature, streamline and vorticity at, $Ra=10^{3}(a_{1},b_{1},c_{1})$, $Ra=10^{4}$ (a_{2} , b_{2} , c_{2}), $Ra=10^{5}$ (a_{3} , b_{3} , c_{3}).



Fig (3).Distributions of temperature, streamline and vorticity at, $Ra=10^6$ (a_1,b_1,c_1), $Ra=10^7$ (a_3,b_3,c_3).



Fig (4). Developing Isothermal Lines at Ra=10³ with Respect to Aspect Ratio.



Fig(5). Developing Isothermal Lines with Respect to Aspect Ratio at Ra=10⁶

4.2 Nusselt Number Variation.

The spatial distribution of the avarage Nusselt number (Nu_{av}) along the hot section depends on the discrete heat source location ,Rayleigh number, and (Ar). Fig(6), shows for low (Ra), the curves maintain a flat trend that means low temperature gradients but(Nu_{av}) increases rapidly with (Ra) especially for Ra>10³. Fig(7) shows the effect of (Ra) and (Ar) on the avarage Nusselt number (Nu_{av}) for a vertical discrete heat source, (Nu_{av}) increases with increasing (Ra) , and decreases with (Ar) ^[6]. In general, the average (Nu_{av}) remains invariant up to a certain value of (Ra) and then increases with increasing (Ra). The average Nusselt number increases with increasing (Ar) ,these variations of the average Nusselt number differ greatly at higher (Ra) and vice versa, from these observations, it can be concluded that the overall heat transfer process improves as the (Ar) increases as shown in Fig(8) ^[9]





Fig (8).Developing average Nusselt Number with Aspect Ratio for vertical and horizontal heat source for different values of Raleigh Numbers.

Fig (9). Average Nusselt Number Variation of vertical Heat Source Compared to Available Correlation. (Al-Bahi, et al., 2002)^[4]

4.3 Numerical Model Validation.

In order to validate the numerical model, the results of (Nu_{av}) and (Ra) are compared with previous works. For vertical heat source the results are compared with $(M, Al-Bahi, et al., 2002)^{[4]}$, Fig(9). There are agreement in results and found to be acceptable which validate the present computational study.

5. Conclusions.

Natural convection in enclosure with a single discrete heat source in vertical and horizontal wall, air is the working fluid, studied with the adiabatic upper and lower walls. The partial differential equations for two dimensional conservation of mass, momentum and energy are solved based on finite difference method. The model is then used to investigate the effect of hot strip location on both heat transfer and fluid flow patterns, within the cavity for fixed heater length ℓ/W , $\ell/L = 0.33$ and different positions, S/W, S/H= 0.17. The present data shows in general highest heat transfer rate on the average Nusselt number of the discrete heaters tends to improve with the increase of the Raleigh numbers, the heat source on the horizontal wall can be more active than that on the vertical wall^[12].

The effect of enclosure aspect ratio, for horizontal heat source increasing of average Nusselt number with increasing Ar, and for vertical heat source there is a decreasing in average Nusselt number with increasing Ar.

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