# Performance Evaluation of Selected Serial Concatenation Codes in AWGN & Frequency Selective Rayleigh Fading Channels

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# Abstract

Concatenation codes have attracted a great deal of interest since their discovery. They have been successfully used in digital communication systems in order to achieve reliable transmission on a noisy channel. In this paper, three models of concatenation codes are designed and tested over AWGN and frequency selective Rayleigh Fading (FSRF) channels. The first model uses convolution codes as outer and inner codes. The second model uses convolution code as outer code and Reed-Solomon code as inner one. While in the third model, Reed-Solomon code is used as outer code and convolution code as inner one. The performance comparisons among these models show that the last model gives the better performance in both of the transmission channels considered and for different types of modulation. It attains a maximum gain of (3dB) in SNR over other models in AWGN channel while attains (8 dB) gain for FSRF channel case when bit error rate (BER) equals to 8\*10<sup>-3</sup>.

الخلاصة

اكتسبت الشفرات المتسلسلة قدرا كبيرا من الاهمية منذ اكتشافها. وقد تم استخدامها بنجاح في انظمة الاتصالات الرقمية لتحقيق ارسال كفوء في القنوات الضوضائية. في هذا البحث ، تم تصميم ثلاثة نماذج من الشفرات (convolution) المتسلسلة وتم فحص ادائها في قناتي AWGN و FRSF. النموذج الاول يستخدم الشفرة الالتفافية (convolution) ممشفر خارجي و شفرة الالتفافية (convolution) كمشفر خارجي و داخلي .النموذج الثاني يستخدم الشفرة الالتفافية (Reed Solomon) المتسلسلة وتم فحص ادائها في قناتي AWGN و FRSF. النموذج الاول يستخدم الشفرة الالتفافية (Reed Solomon) كمشفر خارجي و شفرة ال (Reed Solomon) ممشفر دارجي و داخلي .النموذج الثاني يستخدم الشفرة الالتفافية كمشفر خارجي و شفرة ال (Reed Solomon) كمشفر دارجي و شفرة ال (Reed Solomon) استخدمت كمشفر دارجي و الشفرة الالتفافية كمشفر دارجي و شفرة ال (Reed Solomon) كمشفر دارجي و شفرة ال (Reed Solomon) استخدمت كمشفر خارجي و الشفرة الالتفافية كمشفر دارجي و شفرة ال (Reed Solomon) كمشفر دارجي و أنفرة الالتفافية كمشفر دارجي و شفرة ال (Reed Solomon) استخدمت كمشفر خارجي و الشفرة الالتفافية كمشفر دارجي . بينما في النموذج الثالث فأن شفرة ال (Reed Solomon) استخدمت كمشفر خارجي و الشفرة الالتفافية كمشفر داخلي . بينما في النموذج الثالث فأن شفرة ال (Reed Solomon) استخدمت كمشفر خارجي و الشفرة الارسال كمشفر داخلي . بينما في أنا مقارنة الأداء الثلاثة بينت أن النموذج الأخير يعطي الأداء الأفضل في كل من قناتي الأرسال المعتبرتين و لأنواع التضمين المختلفة وأنه حقق أقصى كسب 3 ديسي بيل من نسبة الأشارة الى الضوضاء بالنسبة وهنبرة الأخرى في قناة AWGN بينما حقق كسب 8ديسي بيل من نسبة الأشارة الى الضوضاء بالنسبة وهنبرة النادج الأخرى في دالة بنا وهدي المعن المعنون المعنين الموضاء مالنسبة وهدين المعنون المغرمين الماذج الموضاء بالنسبة وهدي الموضاء الموضاء الموضاء والنسال الموضاء والنسبة الأشارة الى النموضاء والنسبة وهذي المي الموضاء في حالة قناة AWGN وهذين الكسبين قد تصبة في المنادج الأخرى في دالة الموضاء الساوى قاله المعان الموضاء الموضاء في حالة الموضاء وولين الموضاء الموضاء في حالة الموضاء الموضاء وولين الموضاء وولي النموضاء في حالة الموضاء الموضاء الموضاء وولي الموضاء الموضاء الموضاء الموضاء وولي الموضاء الموضاء وولي الموضاء الموضا

## 1. Introduction:

Concatenation of codes is a very useful technique that leads to the construction of very efficient codes by using two or more constituent codes of relatively small size and complexity. Thus, a big, powerful code with high BER performance, but of impractical complexity, can be constructed in an equivalent concatenated form by combining two or more constituent <sup>[1-3]</sup>. The concatenation code can be realized by combining two codes: inner and outer, separated from each other by an interleaver either in serial or parallel fashion. The performance of resulting concatenation code depends on the type of inner and outer codes used weather they similar or different especially for serial concatenation version.

In the previous works <sup>[2-6]</sup>, the effect of changing the type inner and outer codes was studied, but the effect of reordering the type of inner and outer codes is no longer being studied. In this paper, the performance of three serially concatenation models are presented. The first model consists of two convolution codes as inner and outer codes. The second model uses convolution code as outer code and Reed-Solomon code as inner one. While the third model consists of Reed-Solomon code as outer code and convolution code as inner one. The performances of the three mentioned models are tested over AWGN and frequency selective Rayleigh fading channels. For convolution code, Viterbi decoding algorithm is used while Berlekamp–Massey algorithm is used for Reed-Solomon code.

The rest of the paper is arranged as follows. In section 2, a description to the proposed concatenation code is presented. The simulation results and its discussion are given in section 3. Finally, the most important conclusions drawn from work are discussed in section 4.

## 2. System Description:

The structure of the implemented serially concatenation code (SCC) is shown in Figure (1). Three models are implemented according to this structure. The first model consists of a two convolution codes, the outer code  $C_o$  with code rate  $R_o=1/2$  with m=2, where m is the number of memory element and inner code  $C_i$  is also with rate  $R_i=1/2$  and m=2, joined by an interleaver of length *N* bits, generating an SCC with rate  $R_c = R_o * R_i^{[2]}$ . Note that *N* must be an integer multiple of the outer encoder length [1].

The inner and outer encoder of the convolution code  $C_o$ ,  $C_i$  are represented by the generator polynomials in octal form G = [5 7], and the minimum free distance DF=5, so that this code is able to correct two error t=2. The received signal is decoded by inner and outer decoder with a hard decision of the Viterbi algorithm (VA) which performs the maximum likelihood decoding <sup>[7]</sup>.

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Figure (1) Serial Concatenation Code With Interleaver System.

The basic goal of using an interleaver in the above models is to randomize the data sequence. When used against burst errors, interleavers are designed to convert error patterns that contain long sequences of serial erroneous data into a more random error pattern, thus distributing errors among many code vectors. Burst errors are characteristic of some channels, like the wireless channel, and they also occur in concatenated codes, where an inner decoder overloaded with errors can pass a burst of errors to the outer decoder. Here a block interleaver is used, so the data are first written in row format in a permutation matrix, and then read in column format <sup>[7]</sup>.

Maximum-likelihood decoding of convolution codes is equivalent to the computation of a shortest path on a particular directed graph called a *trellis*. A trellis node is labeled with a pair(s,t) where s represents the state of the encoder at time t. An edge  $(s,t) \rightarrow (\bar{s},t+1)$  in the trellis represents the transition of the encoder at time t from state (s,t) to state  $(\bar{s},t+1)$ . Each edge $(s,t) \rightarrow (\bar{s},t+1)$  in the trellis is labeled with a nonnegative branch metric d which measures the likelihood that the encoder moves into state  $\bar{s}$  at time t+1 given that the encoder is in state s at time t+1 and given the received symbol at time t.

The branch metrics can be defined in such a way that the sum of the branch metrics on a path is a measure of the likelihood of that path. A trellis contains a distinguished *start node* at time 0. The *accumulated metric* of a node is the distance of the node from the start node. The goal of the decoder is to identify, for each time step t, the node at time with the smallest accumulated metric. The Viterbi algorithm maintains an upper bound to the accumulated metric of all nodes. The basic operation is the *expansion* of a node: i.e. the accumulated metric of a node is known, the upper bound of all its successors is updated. The Viterbi algorithm expands nodes breadth-first, and it expands the whole trellis no matter what the noise conditions are <sup>[8]</sup>.

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The second model consists of two codes, the outer code is the convolution code with code rate of  $R_0=1/2$ , and with m=2 and the inner code is Reed-Solomon code with a code rate of  $R_i = 5/7$ , this code is based on Galois field of 8 GF(8) each element of three bit, it is able to correct any error pattern of size t = 1, and its generator polynomial is <sup>[7]</sup>:

where  $\alpha$  is a primitive element in GF (8), the received signal is decoded in the inner decoder by Berlekamp–Massey Algorithm<sup>[7]</sup> then the decoded signal is deinterleaved, so that some of the errors may get corrected, the output of the deinterleaver entered to the Viterbi decoder to correct the remaining errors accordingly to the correction capability of the outer code.

Finally, the third concatenation model consists of Reed-Solomon code (7, 5) code as outer code and convolution code with  $R_i = 1/2$  as inner one.

The transmitted signals in all proposed concatenation coding models are transmitted through two types of channels: AWGN and FSRF channels. For AWGN channel, noncoherent BFSK and QPSK modulation are used while in frequency selective Rayleigh fading noncoherent BFSK modulation is used. Noncoherent BFSK modulation is used channel because it is an attractive modulation when the phase changes too quickly to be tracked which is caused by the characteristic of FSRF channel <sup>[9]</sup>.

This channel is characterized by two different types of distortion: the first is the frequency selectivity which depends on the multipath spread  $T_m$  or equivalently on the coherence bandwidth. To specify the degree of frequency selectivity, a parameter that is called effective delay spread ratio  $\mu=T_m/T_b$  is used. The other type of distortion depends on the time variation of the channel, which are characterized by the coherence time  $(\Delta t)_c$  or, equivalently, by the Doppler spread  $B_d$ . The multipath time delay spread is assumed to be less than the bit duration  $T_b^{[10]}$ .

## 3. Simulation Results:

The performances of the three concatenation codes described in section II are tested in AWGN and FSRF channels in terms of bit error rate (BER) versus signal to noise ratio (SNR). The simulation parameters used for the codes are the same as in section II. For frequency selective Rayleigh fading channel case: two paths are considered with effective delay spread ratio  $\mu$ =0.1and coherence time ( $\Delta t$ )<sub>c</sub> = 5 and 10 seconds. The simulation tool used was MATLAB code ver.7.

#### **3-1 AWGN Channel Results:**

Figures (2) and (3) illustrate the bit error rate for the AWGN channel with QPSK modulation and FSK modulation for the three concatenation code models (denoted as **conv,conv, conv,RS** and **RS,conv**, where **conv** stands for convolution code and RS stands for Reed-Solomon code) and uncoded case. It is clear from these figures that the performance of the third system (RS,conv) is superior than others in both BFSK and QPSK modulation cases. For example a 1 dB gain in SNR is obtained when BER  $=8*10^{-4}$  over the second model (conv,RS) in QPSK case and 3dB and 4dB in BFSK case over the first (conv,conv) and second models respectively.

However, the performance of the first and third model is approximately same in QPSK case. Another point can be concluded from these figures is that the improvement in BER over the uncoded case starts at SNR=0 dB for all models in QPSK modulation. In BFSK, only the third model start to improve BER at SNR=0 dB, while the improvement in other models start at about SNR=3 dB. This means that making RS code as an outer code offers the best concatenation code performance in AWGN under our set conditions.



Figure(2) Performance Of SCC With QPSK Modulation In AWGN Channel.



Figure (3) Performance of SCC with FSK modulation in AWGN channel.

### 3-2 Frequency Selective Rayleigh Fading Channel Results:

Figures (4) and (5) illustrate the bit error rate for the frequency selective Rayleigh fading channel with noncoherent detection of BFSK modulation for the three concatenation code models. These figures show that the third model has the better performance as compared with others. For example at BER=  $8*10^{-3}$ , a gain of 8 dB in SNR is obtained over the first and second models when  $(\Delta t)_c = 5$  sec, while 4dB and 6dB gains are obtained over the first and second models respectively when  $(\Delta t)_c = 10$  sec. This means that the improvement introduced by the third model over the other models and the uncoded case increases as coherence time of FSRF increases. It is also noted from Figure (4) that performance of the first and the second models become similar to the uncoded case for SNR values greater than 16dB.

One justification of why the third model offers a superior performance than other models is that the Convolutional codes are sensitive to burst errors and, when they fail to decode properly, often produce extended error bursts. RS codes, on the other hand, work well with burst errors, so the bursty errors produced by the inner convolution code can often be corrected by the outer RS codec. A row-column interleaver which is used here is a subclass of block interleaver which has the

#### Figure (4) Performance of SCC with BFSK modulation over FSRF channel,

 $(\Delta t)_c$  =5 sec.



Figure (5) Performance of SCC with BFSK modulation over FSRF channel,  $(\Delta t)_c = 10$  sec.

Advantage that if the data packets are sized to fit within an interleaver block, no flush is required. The drawback to the block interleaver is that it is only possible to synchronize at interleaver block boundaries.

# 4. Conclusions:

The performance of serial concatination code depends on the type of inner and outer codes used and on the order arranging them within the concatination opertaion. For the codes types used in this paper, putting the Reed-Solomon case as outer code and convolution code as inner one offers the best performance as compared with other arrangements.

The gain obtained in terms of dBs in SNR for the best concatination arrangement in FSRF test channel was greater than that obtained for AWGN test channel. This means that the proposed concatination code can be used efficiently for channels under sever working conditions.



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