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# *Performance Evaluation of Wavelet Packet Based Multicarrier Modulation (WPMCM) System with PAPRs Reduction over Frequency Selective Rayleigh Fading HF Channel*

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## Abstract

***The effectiveness of Wavelet Packet based Multicarrier Modulation (WPMCM) with $\pi$/4-DQPSK modulation is investigated, for communication over HF channel. The simulation results show that the performance of the proposed WPMCM system is better than that of Orthogonal Frequency Division Multiplexing (OFDM) for the same environment.***

***Utilizing the advantage of concentrating energy to certain subspaces of the wavelet packet transform (WPT), a peak-to-average power ratio (PAPR) reduction method for WPMCM systems is introduced in this paper.***

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## الخلاصة

***في هذا البحث تم التحقق من فعالية منظومة تضمين متعددة الناقل المعتمدة على حزمة المويجة (WPMCM) مع أستخدام تقنية التضمين ($\pi$/4-DQPSK) للانصال عبر فناة متغيرة مع الزمن مثل القناة الراديوية عالية التردد (HF channel). أثبتت نتائج البحث ان المنظومة المقترحة (WPMCM) أعطت نتائج افضل من منظومة التعدد التقسيمي الترددي المتعامد (OFDM) لنفس الظررف.***

***بالاستفادة من تركيز الطاقة في أجزاء من تحويلة حزمة المويجة (WPT),تم استخدام طريقة تقليل نسبةقيمة القدرة الى المعدل(PAPR) في منظومة (WPMCM) في هذا البحث.***

## 1- Introduction

Multicarrier modulation (MCM) [1] based on the discrete Fourier transform (DFT) has been adopted as the modulation/demodulation scheme of choice in several digital communications standards. These include wireline systems such as digital subscriber lines (DSL), where it is commonly known as discrete multitone (DMT), and wireless systems such as digital audio and terrestrial video broadcast (DAB/DVB-T), local area networks (IEEE 802.11a/g/n), and metropolitan area networks (IEEE802.16a), where it is commonly known as orthogonal frequency-division multiplexing (OFDM).

OFDM splits a high rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of overlapped subcarriers. These subcarriers are modulated with subcarriers spacing, which are selected such that modulated subcarriers are orthogonal over symbol duration. Increasing symbol duration will result in a lower rate parallel subcarriers. This decreases the relative a mount of dispersion in time caused by multipath delay spread. Intersymbol interference (ISI) is eliminated almost completely by introducing a guard time in every OFDM symbol. In guard time, the OFDM symbol is cyclically extended to combat the frequency selective of the channel and to avoid intercarrier interference (ICI) [2].

In recent years, wavelet bases have been introduced into the communication field as an alternative approach to MCM. Wavelet packet based MCM (WPMCM) is a carrierless that uses a filtering and defiltering technique to convey orthogonal multi-sub-band information from transmitter to receiver. WPMCM shares all the benefit of Multicarrier technique and exhibits further benefit such as higher efficiency due to elimination of guard interval (GI). It is considered as one of wavelet transforms which are well localized both in time and frequency domain, while sinusoid waveforms, are only localized in frequency but not in time domain. The wavelet and subband transform applications in communications are viewed in [3]. In [4], the performance of wavelet modulation (WM) in additive white Guassian noise and Rayleigh fading channel is studied. The comparison of binary phase shift keying (BPSK) and WM in a frequency selective channel is performed.

Jamin and Mahonen study the performance of wavelet packet transform modulation (WPM) for transmission over wireless channels [5]. This scheme is shown to be over all quite similar to OFDM, with some interesting additional features and improved characteristics. A detailed analysis of the system's implementation complexity is reported. In [6], a novel signal processing method based on wavelet packet analysis is proposed to combat the adverse communication environment over power lines.

A comparison research between wavelet transform and wavelet packet transform is presented. Since wavelet packet decomposition can provide more precise frequency resolution than wavelet decomposition, this paper proposes to use the wavelet packet analysis to deal with the highly polluted PLC communication signal.

The following additional attractive features inherited from the wavelet packet modulation: The Wavelet Transform (WT) is widely adopted in image/video and speech coding. It use for modulation/demodulation on frequency-selective channels results in a better integrated system design and a reduced overall implementation cost. The WPT uses only real arithmetic, as opposed to the complex-valued DFT. This reduces the signal-processing complexity/power consumption.

## 2- Construction of wavelet bases

The wavelet transform is a kind of technique derived from the Fourier transform. The most important difference between these two transformations is that individual wavelet functions are localized in space, while Fourier sine and cosine functions are not. This localization feature of wavelet, along with wavelets' localization of frequency, provides lots of special characteristics. This makes wavelet transforms different from Fourier transforms. It can provide main sidelobes of much lower magnitude than those of Fourier transforms. This is also one of the reasons why we have used wavelet bases to modulate symbols in MCM systems. In the following, let $Z_+$ denote the set of nonnegative integers, i.e., $Z_+ = \{0, 1, 2, \ldots\}$. The wavelet packets are defined recursively by a sequence of functions using pairs of quadrature mirror filters (QMFs) h(k) and g(k) of length F [7]:

$$w_{2n}(t) = \sqrt{2}\sum_{k \in Z_+} h(k) w_n(2t-k), \tag{1}$$

$$w_{2n+1}(t) = \sqrt{2}\sum_{k \in Z_+} g(k) w_n(2t-k), \quad n \in Z_+ \tag{2}$$

The sequences h(k) and g(k) correspond to the discrete impulse response of a QMF bank with perfect reconstruction, where the relationship $g(k) = (-1)^k h(1-k)$ holds. The function $w_0(t)$ is the unique fixed point of the first two-scale equation obtained from Equation (1) when $n = 0$. It is exactly the scaling function from a multiresolution analysis (MRA). Similarly, $w_1(t)$ is the corresponding wavelet function from Equation(2). The elements of the set $\{w_n(t)\}_{n \in N}$ are called the wavelet packet functions, which have two useful properties. Denote $\langle x, y \rangle$ the integral operator, then [6] :

$$\langle w_n(x-j), w_n(x-k) \rangle = \delta_{j,k} \qquad j,k \in Z_+, \tag{3}$$

$$\langle w_{2n}(x-j), w_{2n+1}(x-k) \rangle = 0 \quad j,k \in Z_+ \tag{4}$$

Equation (3) states that each individual wavelet packet function is orthogonal with all its nonzero translations. This property will be utilized to eliminate ISI. Equation (4) states that every pair of packet functions from the same parent packet are orthogonal at all translations. Therefore, $\{w_n(t)\}_{n \in N}$ is a set of orthogonal functions. In WPMCM, the baseband sequence $\{c_k\}$ is obtained from the wavelet reconstruction algorithm according to:

$$c_k^{(j)} = \frac{1}{\sqrt{2}} \sum_{\ell \in Z_+} \left[ g(n-2\ell) c_\ell^{(j-1)} + h(n-2\ell) d_\ell^{(j-1)} \right] \tag{5}$$

where $\{c_k^{(j)}\}$ and $\{d_k^{(j)}\}$ are the kth symbol of the jth subband. The transmitted baseband signals are again transformed into subband signals using the wavelet decomposition algorithm. Thus,

$$c_k^{(j-1)} = \frac{1}{\sqrt{2}} \sum_\ell r(2k-\ell) c_\ell^{(j)} \tag{6}$$

$$d_k^{(j-1)} = \frac{1}{\sqrt{2}} \sum_\ell \eta(2k-\ell) c_\ell^{(j)} \tag{7}$$

where $\{r_k\}$ and $\{\eta_k\}$ are the decomposition sequences of the wavelet. By choosing different $\{r_k\}$ and $\{\eta_k\}$, we can get different kind of wavelets.

Figure(1) gives the decomposition and reconstruction process of two-level wavelet functions. The symbol "$\downarrow$" means decomposition and "$\uparrow$" means reconstruction.

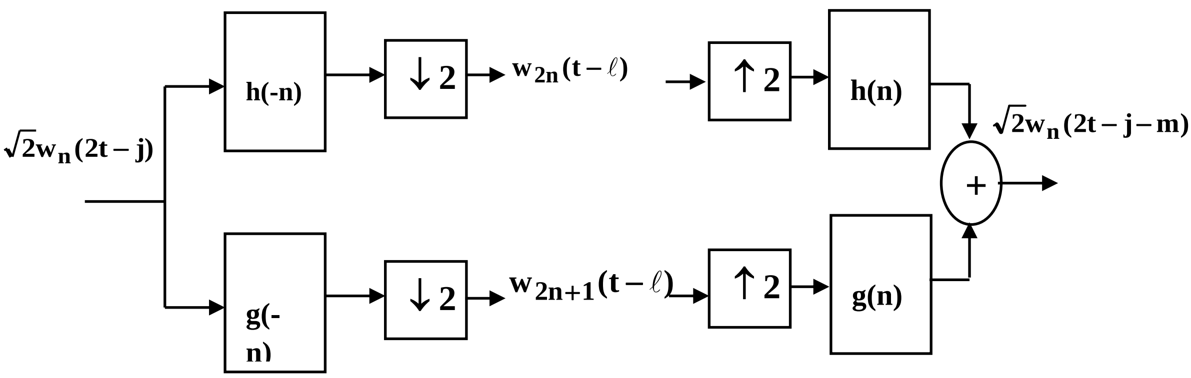



***Figure (1) The decomposition and reconstruction of wavelet functions***

## 3- Wavelet Packet –Multicarrier Modulation (WPMCM) System

Figure (2) illustrates a generic block diagram of WPMCM transceiver, while the details are shown in Figs. (2) and (3) respectively. WPMCM system employs two filter banks i.e. Inverse Wavelet Packet Transform (IWPT) (reconstruction) placed at the transmitter side, and Wavelet Packet Transform (WPT) (decomposition) placed at the receiver side. The block "MAKE CMPLX" accepts two N-dimensional real vectors as inputs. Its output is an N-dimensional complex vector whose ith complex element is formed from the ith real elements of the two input vectors.

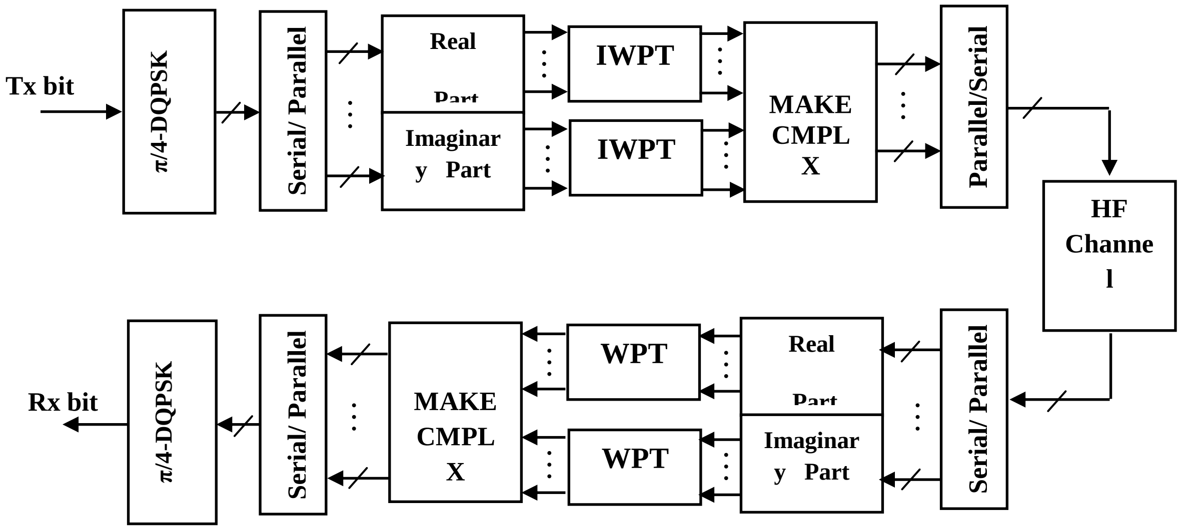



***Figure (2) Block diagram of the WPMCM system model. Arrows with a strike indicate complex quantities.***

## 4- Peak to Average Power Ratio (PAPR) Reduction Method

Like as all the multicarrier modulation the WMCM has large PAPR. As it is known, if the transmitted peak power is limited, no matter by regularity or application constraint, the average power allowed by MCM will be reduced. This will in turn reduce the transmission range of MCM systems.

Thus, to maintain spectral efficiency, a linear amplifier with a large dynamic range is needed. This would degrade the power efficiency greatly, which should be avoided. Therefore, to increase the efficiency of wavelet based MCM systems, methods are required to reduce the high PAPR to economize the power consumption. The discrete wavelet transform (DWT) is a type of batch processing, which analyses a finite length time domain signal by breaking up the initial domain in two parts: the detail and approximation information.

The approximation domain is successively decomposed into detail and approximation domains. We use the properties of the discrete wavelet transform that the DWT is scattered. This means only few coefficients of DWT dominates the representation. This property is widely used in image processing, such as wavelet de-noising.

Using this properly in WPMCM systems, we can reduce the PAPR with little reconstruction loss. The theoretical analysis is as follows. Figure (3) shows the composite threshold method to the transmitter of a WPMCM system.

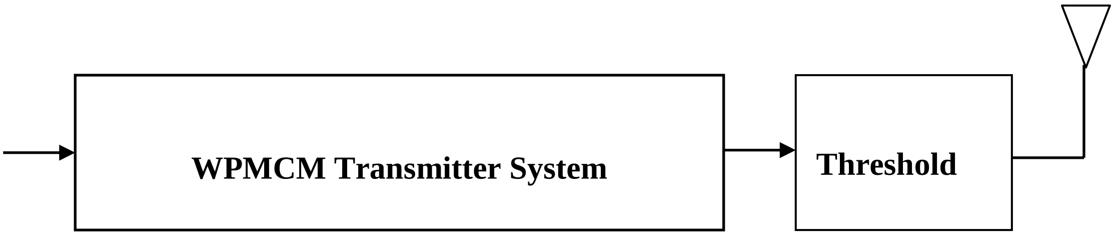



***Figure (3) Block diagram to show threshold of a composite WPMCM waveform***

Let x(n) be the signal obtained after orthogonal modulation. Then, the PAPR can be defined as [7]:

$$\mathrm{PAPR}_{(dB)} = 10\log_{10}\frac{\max_n\{|x(n)|^2\}}{E\{|x(n)|^2\}} = \frac{\max\left\{\left|x(n)^2\right|, n=0,1,\ldots,N-1\right\}}{\frac{1}{N}\sum_{n=0}^{N-1}|x(n)|^2} \tag{8}$$

Since wavelet transforms always concentrate energy on some given number of bases, we can introduce a threshold T and compare it with the energy of each orthogonal base. Then, we define a sequence $x_T(n)$ as:

$$x_T(n) = \begin{cases} 0 & \text{if } |x(n)|^2 < T \\ x(n) & \text{if } |x(n)|^2 \geq T \end{cases} \tag{9}$$

The threshold value (T) can be determined by [7]:

$$T = \mathrm{MAD}(|x|^2)/\sqrt{3\log_2 N} \tag{10}$$

where MAD( . ) is median value of a vector, and N is subcarriers number .

Assume that there are M basis functions whose energies are smaller than the threshold T. Then, let us define a new sequence $x_1(n)$:

$$x_1(i) = x_T(n), \qquad x_T(n) \neq 0, \quad i = 0.1,\ldots,N-M-1; \quad n = 0,1,\ldots,N-1 \tag{11}$$

Reconsidering (8), the PAPR can now be written as:

$$\mathrm{PAPR}_N = 10\log_{10}\frac{\max\left\{|x(n)|^2, \text{for } n = 0,1,\ldots,N-M-1\right\}}{\frac{1}{N-M}\sum_{n=0}^{N-M-1}|x(n)|^2} \tag{12}$$

Let the threshold T fulfill the inequality $T < \frac{1}{N}\sum_{n=0}^{N-1}|x(n)|^2$ .

From Equations (10), (11), and (12), we obtain, $PAPR > PAPR_N$.

One more important measurement, which can be helpful in analyzing the performance of a multicarrier modulation system, is Cumulative Distribution Function (CDF) of PAPR. CDF is the probability of the PAP ratio that is below some threshold level and can be written as [8]:

$$CDF = Pr(PAPR \leq PAPR_0) \qquad (13)$$

where, ($PAPR_0$) is a threshold value of PAPR.

The PAPR reduction (in dB) is obtained by:

$$PAPR\ Reduction = PAPR\_ - PAPR_T \qquad (14)$$

where PAPR represents the PAPR without threshold, and $PAPR_T$ represents the PAPR after threshold is taken.

## 5- Simulation Results

The proposed system of the WPMCM is shown in Figure (2). The simulation results of this system are obtained by using MATLAB version 7. These results are compared with OFDM over frequency selective Rayleigh fading HF channel environment. Table (1) shows the parameters are used in WPMCM system.

***Table (1) Simulation parameters***

| | |
|---|---|
| Modulation Type | $\pi$/4- DQPSK |
| Number of subcarriers | 32,64,128,256 |
| Channel model | AWGN + HF channel |
| Bit rate | 4800 bps |
| Coherence time $(\Delta t)_c$ | 5 sec |
| Delay spread($T_d$) | 3 msec |
| Number of path | 8 |
| Number of transmission bits | 50000 |

A comparison between the (BER) performance of WPMCM with duabechies of order 6, db6, and OFDM system for $\pi/4$-DQPSK modulation is illustrated in Figures (4) and (5) for N=64 and 128 respectively. OFDM is cyclically extended with guard time, $T_g$=5 msec ($T_g$>$T_d$)[2]. In these figures, the amount of SNR required to obtain BER=$10^{-3}$ for OFDM is larger than that of WPMCM by about 2 dB.

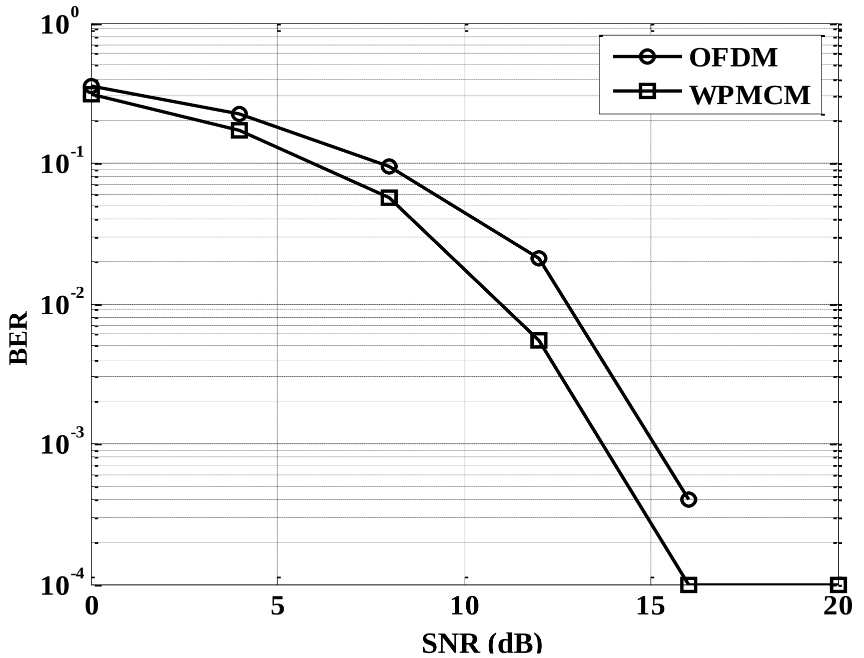



***Figure (4) Comparison of BER performance between WPMCM and OFDM systems for N=64.***

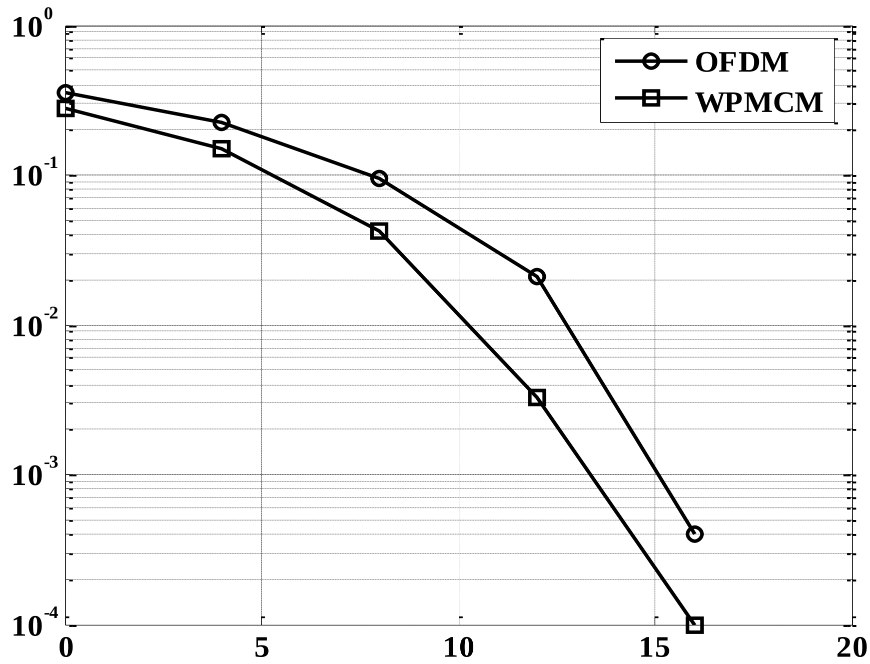



***Figure (5) Comparison of BER performance between WPMCM and OFDM systems for N=128.***

## Effect of subcarrier number:

Figure (6) illustrates BER versus SNR for various number of subcarriers of WPMCM system when db6 is used. From this figure, we observe that when subcarriers number increases, the BER performance improves.

But by increasing the number of subcarriers the peak to average power ratio increases and the implementation complexity also increases.

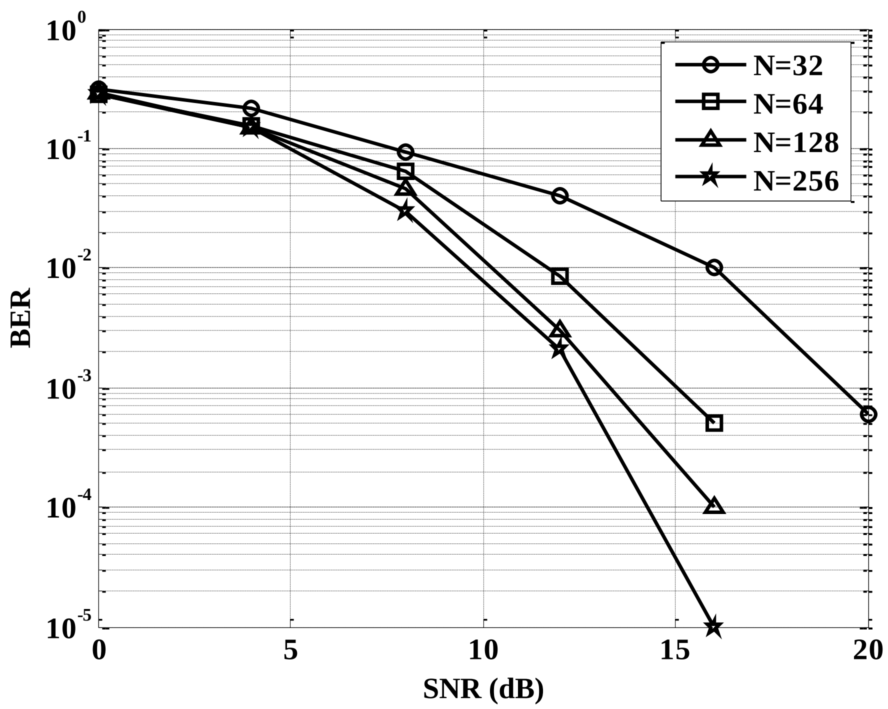



***Figure (6) BER performance of WPMCM system for various number of subcarriers***

## Effect of the wavelet family:

Three wavelet packets from three different families are used for this simulation namely Daubechies wavelet packets with order 6, *db6*, Symmlets wavelet packets with order 6, *sym6*, and Coiflets wavelet packets with order 2, *coif2*.

These families are chosen because they are orthogonal and have compact support (i.e., fast decaying time, which results in good localization in both time and frequency). The filter length is taken to be F = 12. Figure (7) shows BER versus SNR for these three wavelet families for N=128. The BER performance is essentially the same for all these wavelet families.

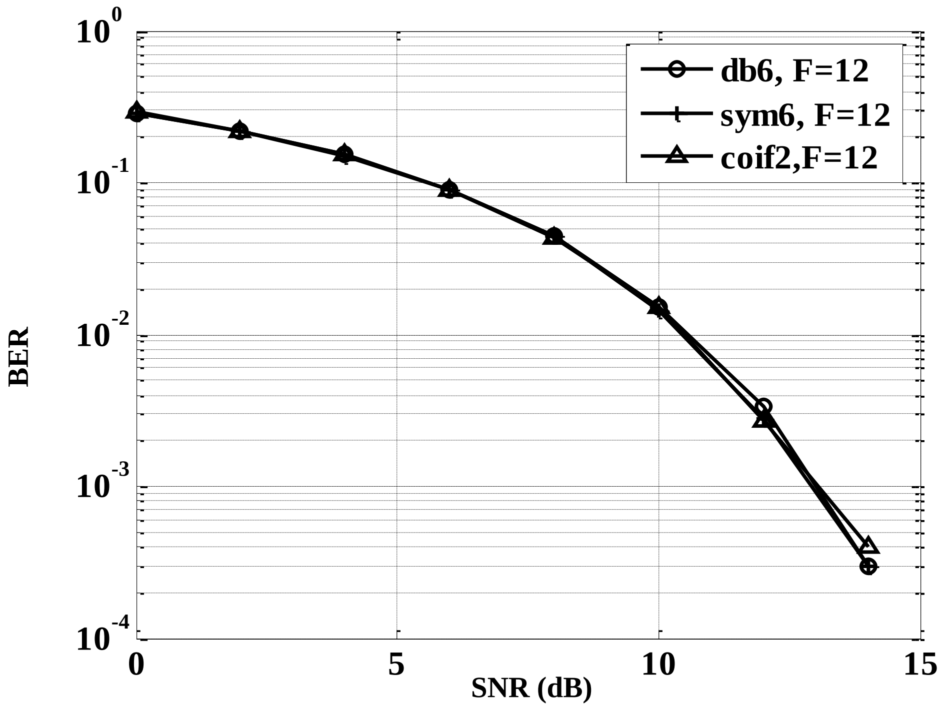



***Figure (7) Effect of wavelet family on the BER performance***

## Effect of filter length:

Figure (8) shows the BER performance as a function of SNR for three wavelet packets from Daubechies family, namely *db3*, *db6* and *db9* which have filter lengths, F=6, 12 and 18, respectively for N=128. From this figure, the performance of wavelet packets with larger filter length becomes better but this improvement is small.

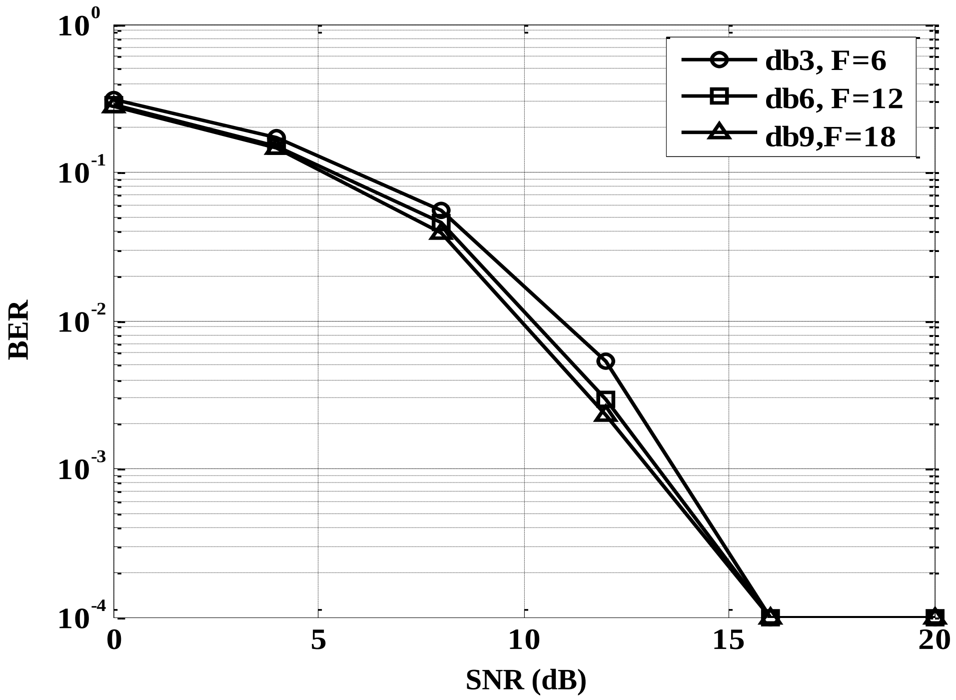



***Figure (8) Effect of scaling filter length on BER performance***

## PAPR results:

Figure (9) shows log of CDF distribution of WPMCM system for different wavelet family (db3, db6, db9, sym6 and coif2) at N=128. This figure shows the Daubechies families approximately have the same distribution of CDF and sym6 has the largest PAPR (PAPR about 10.9 dB for Daubechies family, PAPR about 13.5 dB for coif2 and about 16.4 dB for sym6).

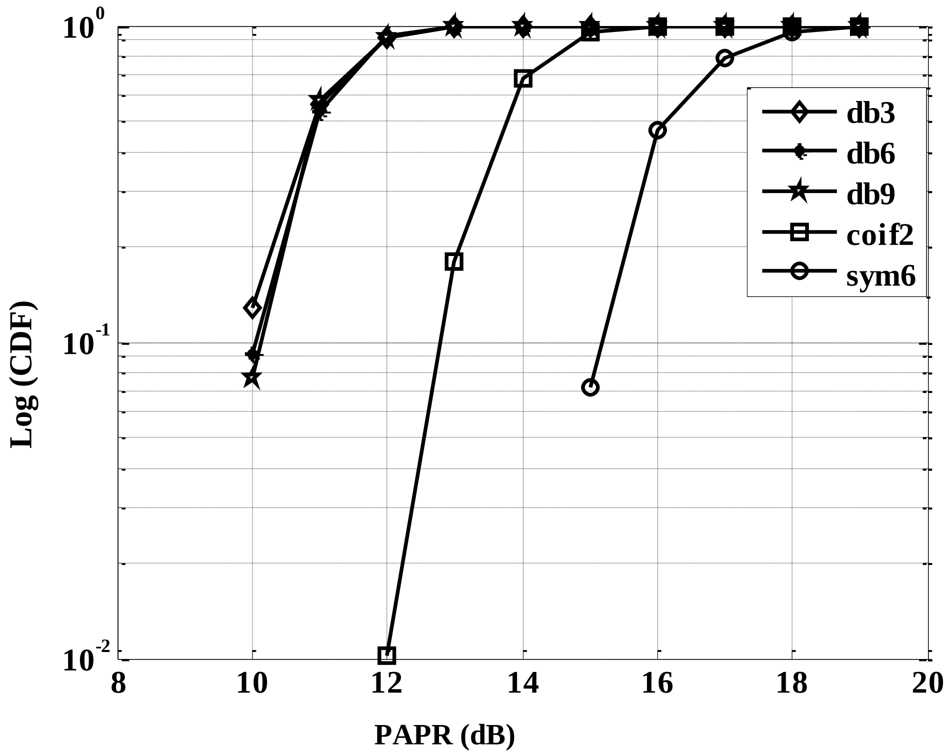



**Figure (9) log of CDF distribution for WPMCM system for different wavelet family.**

Figures (10), (11) show the PAPR Reduction values versus threshold T and BER versus threshold T respectively, for different wavelet family (db3, db6, db9, sym6 and coif2) at N=128 and SNR=20. From these figures, it can be seen that the threshold method can result in significant improvements with respect to PAPR reduction. The range of PAPR reduction enlarges with increasing the threshold T. These figures also show that, the good select of T value is between 0.02 and 0.04 for Daubechies except that db3 that is behaves like coif2, and between 0.04 and 0.1 for coif2 and sym6. Also, sym6 has the largest PAPR Reduction (about 8 dB) with less effect on BER performance.

Figures (12), (13), (14), (15), and (16) show BER performance of WPMCM system for different threshold for db3, db6, db9, sym6 and coif2 respectively at N=128. From these figure, it can see that the proposed threshold T produce PAPR Reduction about 2.5 dB for Duabechies family, for coif2 the proposed threshold T produce PAPR Reduction about 5 dB and for sym6 about 7 dB with degradation in SNR less than 0.5 dB.

Figure (17) shows comparison of log of CDF distribution for different wavelet family (db3, db6, db9, sym6 and coif2) with and without threshold when proposed threshold is used at N=128.

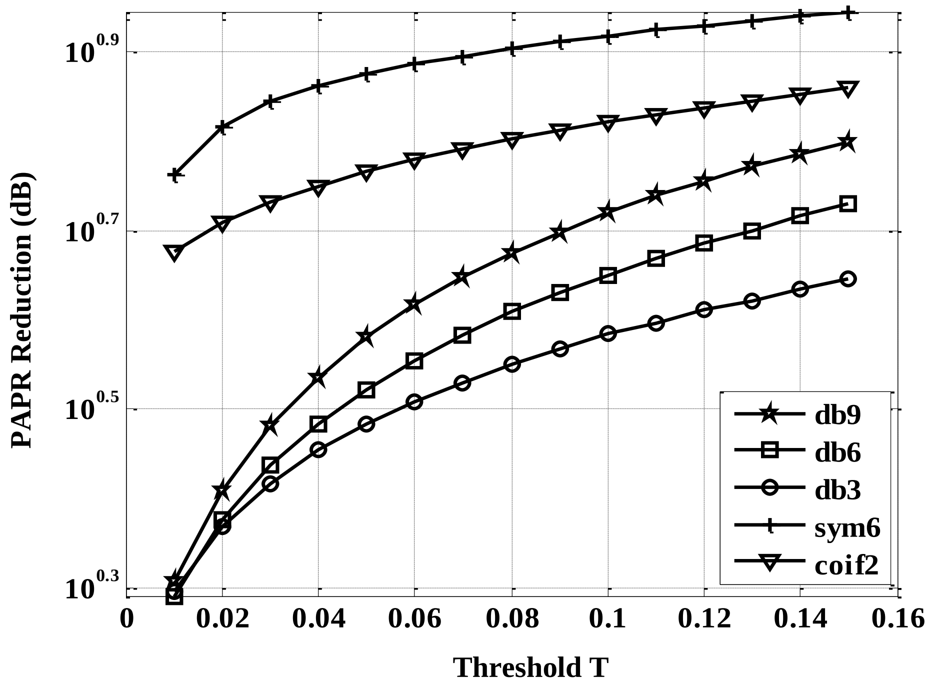



*Figure (10) The PAPR Reduction versus threshold T*

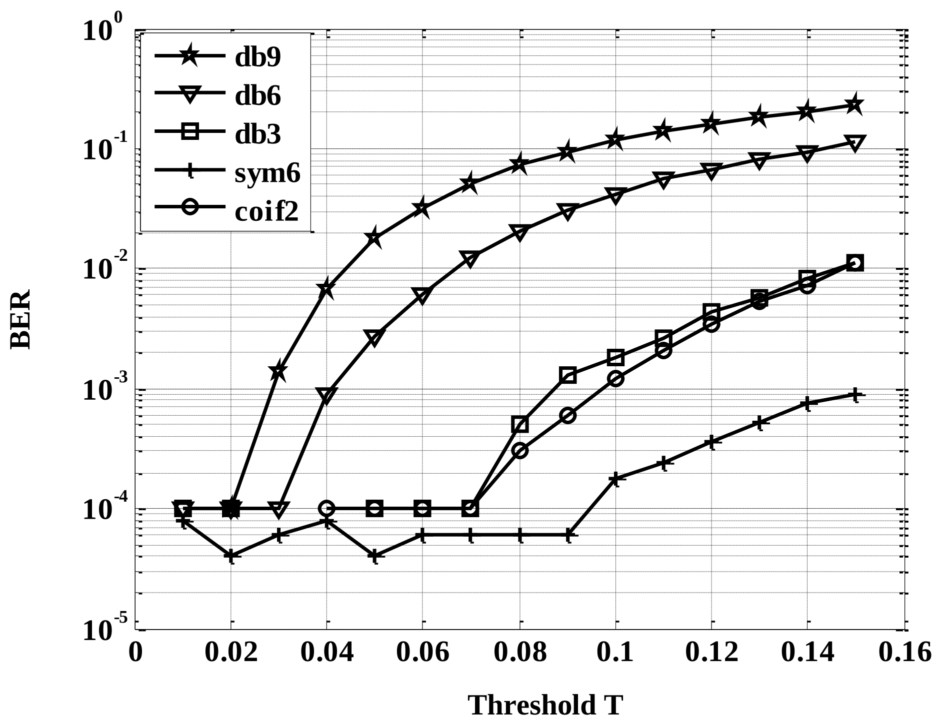



*Figure (11) The BER versus threshold T*

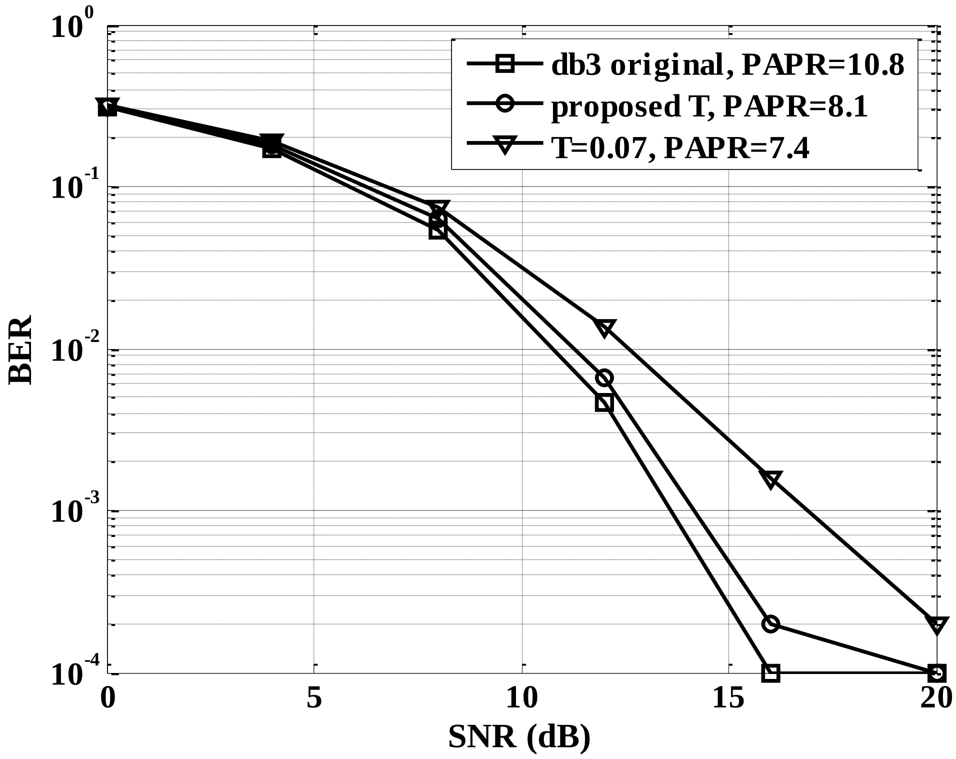



***Figure (12) BER performance of WPMCM system for different threshold T for db3***

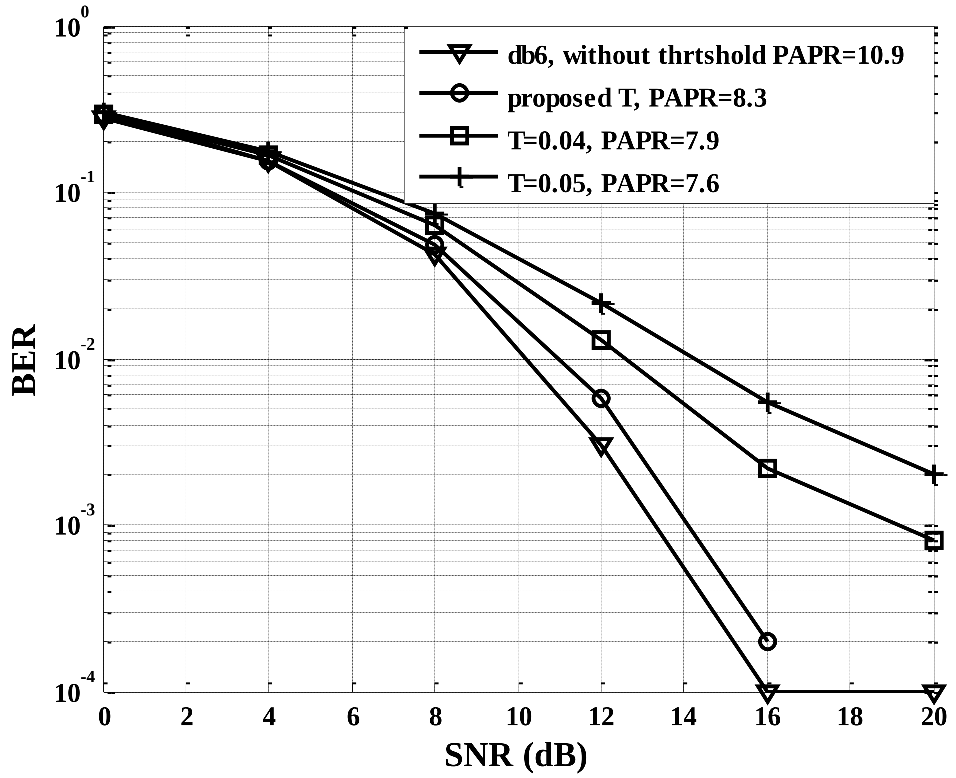



***Figure (13) BER performance of WPMCM system for different threshold T for sym6***

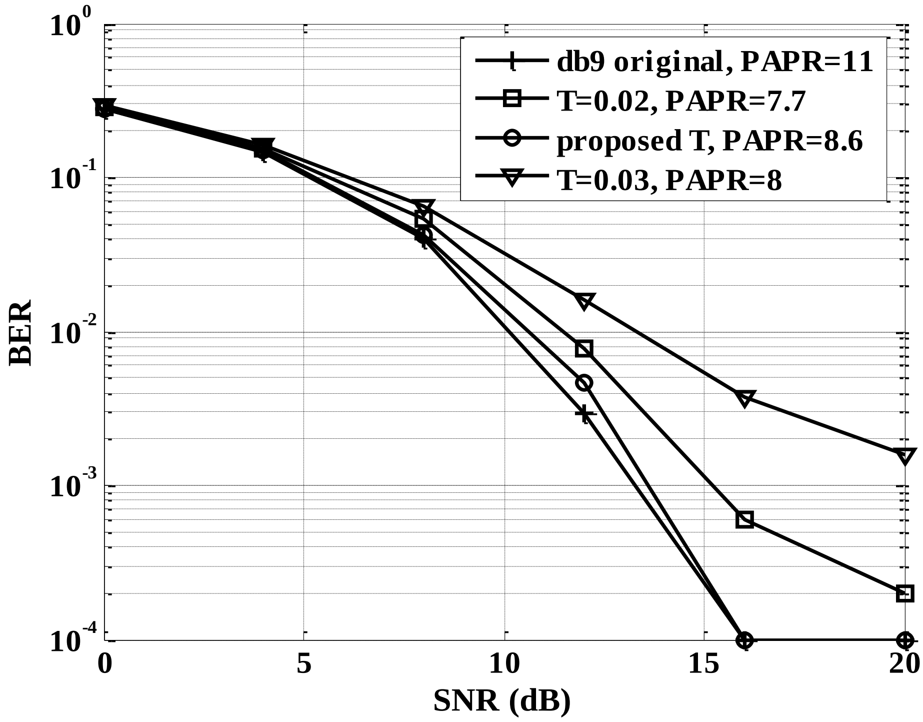



*Figure (14) BER performance of WPMCM system for different threshold T for db9*

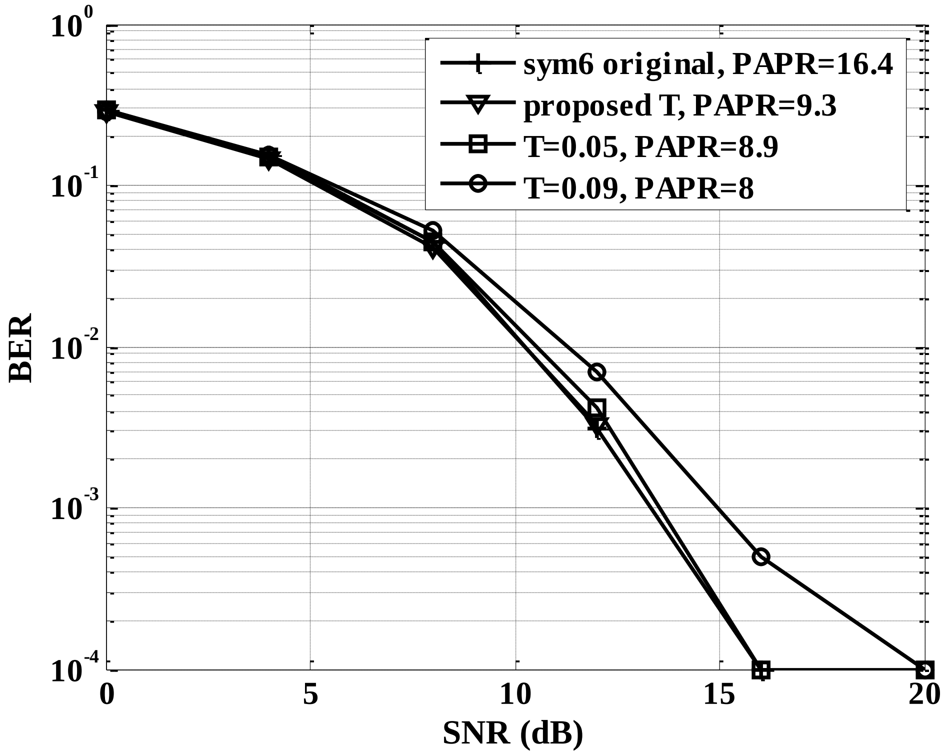



*Figure (15) BER performance of WPMCM system for different threshold T for db6*

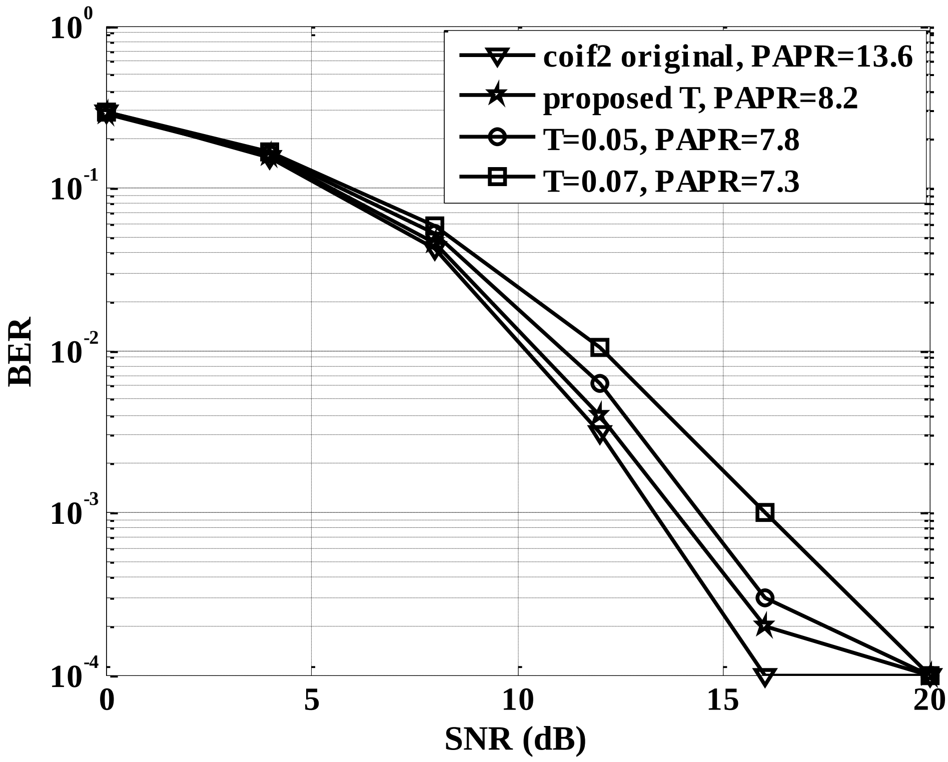



***Figure (16) BER performance of WPMCM system for different threshold T for coif2***

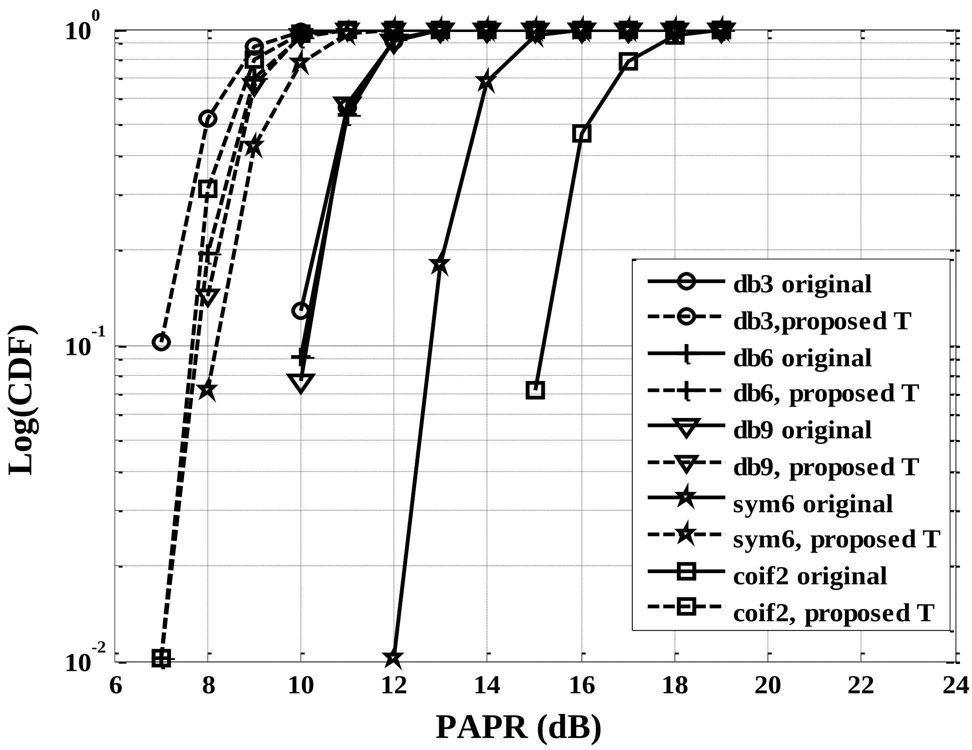
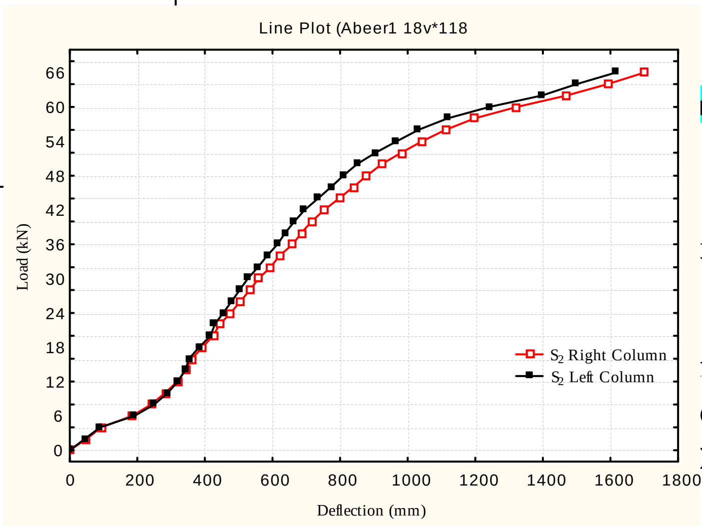



***Figure (17) CDF distribution for different wavelet family with and without threshold.***

## Conclusion

The important points were during simulation and discussions of the results are given below:

- Change wavelet families don't effect on BER performance, but change the length of filter effect more slightly on BER performance.
- CDF gives good estimation for PAPR performance. We can see that from this measure, sym6 has the largest PAPR compared with other families.
- The threshold control PAPR Reduction method can be an efficient way to reduce the PAPR of WPMCM systems. The results show that sym6 and coif2 have the large PAPR Reduction.
- The proposed threshold T produces PAPR reduction of about 2.5 dB for Duabechies family, for coif2 the proposed threshold T produces PAP R reduction about 5 dB and for sym6 about 7 dB with degradation in SNR less than 0.5 dB.



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