# Sidelobe Suppression of Polyphase Coded Signal Using Different Filters

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# Abstract

This paper deals with different types of polyphase coded signals in terms of their correlation properties. The (Frank, P1, P2, P3 and P4) polyphase codes and the Iterative Reweight Least Square(IRLS) procedure were used in this work. Each polyphase code is used as a parameter to (IRLS) procedure and this work is accomplished using MATLAB software. Additive White Gaussian Noise(AWGN) is introduced in the assessment made to the codes that are adopted in this paper. The Peak Sidelobe Level (PSL), the Integrated Sidelobe Level (ISL) and the Signal-to-Noise Ratio Loss (SNRL) criteria are used to evaluate the performance of polyphase codes. It is found that, the P3 code is the best code in terms of (PSL and ISL) criteria within an acceptable (SNRL) when using IRLS procedure. When use AWGN=10dB and for the range P=N till P=N+174, It is found that, PSL of P3 improvement approximately 33dB and ISL of P3 improvement approximately 2dB.

Keywords: polyphase codes, Iterative Reweighted Least Square Procedure, Sidelobes, Peak sidelobes, Integrated sidelobe, Signal to Noise Ratio Loss.

الخلاصة:

يتناول البحث دراسة انواع متعددة من الاشارات ذات التشفير متعدد الاطوار بدلالة خصائصها الترابطية. حيث تم استعمال الشفرات المتعددة الاطوار التالية (P1, P2, P3, P4 and Frank). استعملت طريقة ترشيح التالية اقل مربع تكراري موزون (Iterative Reweight Least Square procedure) وكل شفرة من الشفرات متعددة الاطوار ادخلت الى المرشح وذلك باستخدام برامج مكتوبة بأستخدام برنامج (MATLAB) واضافة ضوضاء من نوع (Gaussian) البيضاء الى الشفرات المستعملة في هذا البحث. لغرض معرفة افضل شفرة يجب استعمال بعض المحددات و التي هي (مستوى اقصى فص جانبي (PSL), مستوى الفصوص الجانبية المتكاملة (ISL), نسبة مستوى المحددات و التي هي (مستوى اقصى فص جانبي (PSL), مستوى الاشارة الى الضوضاء (SNRL)), فستوى الاشارة الى الفوضاء (SNRL)) وحصلنا في المرشح (اقل مربع تكراري موزون) على افضل شفرة هي P3 من ناحية (PSL, ISL) وبقيمة مقبولة لنسبة مستوى الاشارة السمارة السى الضوضاء (SNRL), وعصلنا في المرشح (اقل مربع تكراري موزون) على افضل شفرة مي P3 من ناحية الاشارة الى الموضاء (SNRL) وحصلنا في المرشح (اقل مربع تكراري موزون) على افضل شفرة هي P3 من ناحية (SNRL) وبقيمة مقبولة لنسبة مستوى الاشارة السى الضوضاء (SNRL), وعديما الحية المرشح (اقل مربع تكراري موزون) على افضل شفرة هي P3 من ناحية (SNRL) وبحد ما من SNRL) ومعندما وحدانا من SNRL (المحدم المتخدمة AWGN=10dB وكان مدى ال Pa من Pa الى SNRL تساوي تقريبا PSL لل P3 تحسنت تقريبا SNRL وان ال ISL لل P3 تحسنت تقريبا SNRL وان ال ISL المربع SNRL وان ال ISL المحدمة مقبولة المربع من SNRL (المحدمة SNRL) وحدانا من SNRL (المحدمة SNRL) وجد ان ال SNRL المحدمة تقريبا SNRL (المحدمة SNRL) المحدمة قريبا SNRL (الحدمة SNRL) وان ال ISL المحدمة SNRL (المحدمة SNRL) (ISL (ISL (ISL (SNRL))) وجد ان ال ISL (ISL (ISL (ISL (SNRL)))) وجد ان ال ISL (ISL (ISL (ISL (SNRL)))) وان ال

# 1. Introduction and Literature Survey

Mismatched filter technique is applied to the codes in order to reduce the sidelobes of compressed polyphase-coded signal around the mainlobe, and consequently increase the detection of the received signal. This is done by means of maintain the mainlobe and only reduce the sidelobes. In 1982 B. L. Lewis and F. F. Kretschmer<sup>[1]</sup> were presented two new polyphase pulse compression codes (P3 and P4) and efficient digital implementation techniques. These codes are very Doppler tolerant and that can provide a large pulse compression ratios. The P3 code is not precompression bandwidth limitation tolerant but is much more Doppler tolerant than the Frank or P1 and P2 codes. The P4 code is a rearranged P3 code with better precompression bandwidth limitation tolerance. In 1999 A. J. Zejak, et. al.<sup>[2]</sup> propose a novel method for a potential range resolution improvement, based on the received signal oversampling and the mismatched filter design. In the synthesis of the mismatched filter with improved resolution an algorithm for maximal sidelobe suppression has been used. This idea can be applied in high resolution radar and sonar design.

# 2. Polyphase Codes

Polyphase coded waveforms consisting of more than two phases may also be used in biphase coded waveforms. The phases of the subpulses alternate among multiple values rather than just the 0° and 180° of binary phase codes <sup>[3]</sup>.

### 2-1. Frank Code

The Frank code is derived from a step approximation to a linear frequency modulation waveform. Hence the length of Frank code is equal to N<sup>2</sup>, where N is the number of phases in polyphase codes <sup>[4]</sup>. The phases of the Frank code is obtained by multiplying the elements of the matrix A by phase  $(2\pi/N)$  and by transmitting the phases of row1 followed by row 2 and so on.

$$A = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 2 & \dots & (N-1) \\ 0 & 2 & 4 & \dots & 2(N-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & (N-1) & 2(N-1) & \dots & (N-1)^2 \end{bmatrix} \dots (1)$$

The phase of the *i*th code element in *j*th row of code group is computed as <sup>[4]</sup>

$$Q(i,j) = (2\pi/N) (i-1) (j-1)$$
 ...(2)

where i and j ranges from 1 to N. The output of Matched Filter under zero Doppler, Frank code with length equal to 100 is given in Figure (1).



Figure (1)Frank Code for length 100

### 2-2 The P Codes<sup>[3]</sup>:

The P1 and P2 codes are modified versions of the Frank. The P3 and P4 codes are derived by essentially converting a Linear Frequency Modulation (LFM) waveform to baseband. These tend to be more Doppler tolerant than the Frank, P1, or P2 codes.

#### 2-2-1 P1 Code:

P1 code has been derived from step approximation to linear frequency modulation waveform. The P1 code has  $N^2$  elements and the phase of *i*th element of the *j*th group is represented as

$$Q(i,j) = -(\pi/N) [N - (2j-1)] [(j-1) N + (i-1)] ...(3)$$

where *i* and *j* are integers ranges from 1 to N. The length of the resulting code or compression ratio is  $N^{2}$  <sup>[5]</sup>. The output of Matched Filter under zero Doppler, P1 code with length 100 is given in Figure (2).



Figure(2) P1 Code for length 100

### 2-2-2 P2 Code:

The P2 code has N<sup>2</sup> elements and the phase of *i*th element of the *j*th group is represented as

$$Q(i,j) = (\pi/2N) [N-2i+1] [N-2j+1] \dots (4)$$

where *i* and *j* are integers ranges from 1 to N. The value of N should be even in order to get low autocorrelation sidelobes. The length of the resulting code or compression ratio is  $N^{2}$  <sup>[5]</sup>. The output of Matched Filter under zero Doppler, P2 code with length 100 is given in Figure (3). An odd value of N results in high autocorrelation sidelobes.



Figure (3) P2 Code for length 100

#### 2-2-3 P3 Code:

P3 code has been derived from linear frequency modulation waveform. The phase sequence of the P3 signal is given by <sup>[6]</sup>

$$Q(i) = \pi/N (i-1)^2$$
 ...(5)

Where i varies from 1 to N, and N is the compression ratio. Another two well known polyphase codes are P3 and P4 codes unlike Frank, P1, P2 codes, the length of P3 and P4codes can be arbitrary <sup>[6]</sup>. The output of Matched Filter under zero Doppler, P3 code with length 100 is given in Figure (4).



Figure(4) P3 Code for length 100

#### 2-2-4 P4 Code:

The phase sequence of the P4 signal is given by

$$Q(i) = \pi/N (i-1) (i-N-1)$$
 ...(6)

where i varies from 1 to N and N is the compression ratio <sup>[4]</sup>. The main limitation for implementing the P3 and P4 codes by a structure that implements a Frank code is that the compression ratio must be a square of integers. The output of Matched Filter under zero Doppler, P4 code with length 100 is given in Figure(5). When the P3 and P4 codes are derived by an Extended Frank code, the length of P3 and P4 polyphase codes length can be choose arbitrary <sup>[7]</sup>.



Figure (5) P4 Code for length 100

# 3. Iterative reweighted LS (IRLS) procedure:

The main aim of the mismatched filtering is to form the desired shape of the filter output. In this case the desired shape is, as a rule, a Dirac pulse at the output of compressed filter <sup>[8]</sup>.

**Definition:** suppose the sequence  $s = (s_1, s_2, ..., s_N)^T$  which can be complex, and have to find the coefficients of desired (FIR) filter  $x = (x_1, x_2, ..., x_M)^T$ . The expression then for the filter response (or convolution)  $\Psi = (\Psi_1, \Psi_2, ..., \Psi_{N+M-1})^T$  forms the set of linear equations which may be presented in matrix form

$$\mathbf{S} \mathbf{x} = \mathbf{\Psi} \qquad \dots (7)$$

Where

$$\mathbf{S} = \begin{bmatrix} s1 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ s2 & s1 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots \\ sN & sN - 1 & \cdots & s1 & 0 & \cdots & 0 \\ \vdots & \vdots \\ 0 & 0 & \cdots & sN & sN - 1 & \cdots & s1 \\ \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & \cdots & sN \end{bmatrix}_{(N+M-1) \times M}$$
...(8)

If it is chosen that the filter response  $\Psi$  be equal to the pulse sequence  $\delta$  so that

$$\boldsymbol{\delta} = \begin{cases} \mathbf{0} & \mathbf{i} \neq \mathbf{L} \\ \mathbf{1} & \mathbf{i} = \mathbf{L} \end{cases} \qquad \dots (9)$$

Then the set of (N + M - 1) linear equations with N unknown  $\mathbf{x}_i$  will form

$$\mathbf{S} \mathbf{x} = \boldsymbol{\delta} \tag{10}$$

The relation for least square (LS) filter coefficients estimation can be found which approximates the filter with ideal response <sup>[8]</sup>.

LS: 
$$\mathbf{x} = (S^T S)^{-1} \mathbf{S} \boldsymbol{\delta}$$
 ...(11)

To avoid nonlinear problems in minimax filter design, we proposed the iterative reweighted LS (IRLS) procedure which can be used for real as well as complex codes. Using this procedure we can obtain minimax filter coefficients <sup>[9]</sup>. Iterative procedure:

$$\mathbf{X} = (\mathbf{S}^{\mathrm{T}} \mathbf{R} \ \mathbf{S})^{-1} \mathbf{S} \mathbf{R} \boldsymbol{\delta} \qquad \dots (12)$$

where x are the estimated filter coefficients, and **S** is the signal matrix, having a constant value during the iterative procedure; its structure for the oversampled sequence is as shown in matrix form (8) where  $S_i = (S_{il}, S_{i2}, ..., S_{iN})$  is the matrix formed from the series {Sn}. In Equation (12) *R* is a Diagonal Matrix called the weighting matrix that is equivalent to the Identity Matrix except for the middle element which is zero <sup>[10]</sup>. The desired autocorrelation function which corresponds to the filter response is labeled  $\delta$ , in the design of the improved resolution filter, and is also equal to the Dirac pulse <sup>[2]</sup>.

### 4. The Performance of Sidelobe Suppression Techniques

The following measurement are often used to quantify the performance of range sidelobe suppression techniques:

#### 4-1 Peak Side lobe Level

The peak side lobe level (PSL) of the Autocorrelation Function (ACF) can be defined as <sup>[11]</sup>

$$PSL = 10 \log_{10} \left[ \frac{max R^2(k)}{R^2(0)} \right] \qquad \dots (13)$$

where k is the index for the points in the ACF, R(k) is ACF for all of the output range side lobes except that at k = 0, and R(0) is the peak of the ACF at k = 0.

#### 4-2 Integrated Sidelobe Level

Integrated Sidelobe Level (ISL) of the ACF can be defined as [11]

$$ISL = 10\log_{10} \sum_{k=-M}^{M} \frac{R^{2}(k)}{R^{2}(0)} \qquad \dots (14)$$

#### **4-3 Mismatch Loss**

The Signal-to-Noise Ratio Loss (SNRL) is expressed in decibels as the ratio of the peak output value of the mismatched filter y peak, mismatched, relative to the peak output value of the matched filter

y peak, matched, as follow [11]

$$SNRL = -20 \log_{10} \left[ \frac{y \text{ peak, mismatched}}{y \text{ peak, matched}} \right] \qquad \dots (15)$$

### 5. RESULTS

This section show the result of (PSL, ISL and SNRL)-versus-P with and without noise effect and (PSL, ISL and SNRL)-versus-SNRi.

### 5-1 Zero padding effect on IRLS Procedure In Noise Free Case

Figure (6) shows the PSL-versus-P, P is a Filter length (P=N+ zero padded) and zero padded from (0 to 200). It shows the behaviors of the IRLS procedure when it is applied to five different polyphase codes in noise free case. While, Figure (7) shows the ISL-versus-P behaviors of the IRLS procedure when it is applied to five different polyphase codes in noise free case. Moreover, Figure (8) shows the SNRL-versus-P behaviors of the IRLS procedure when it is applied to five different polyphase codes in noise free case.



Figure (6): PSL-versus-P behavior of the IRLS procedure when it is applied to five different polyphase codes.



Figure (7): ISL-versus-P behavior of the IRLS procedure when it is applied to five different polyphase codes.



Figure (8): SNRL-versus-P behavior of the IRLS procedure when it is applied to five different polyphase codes.



(9-e)Frank code, SNRi=10dB.

Figure (9): The normalized magnitude output of the IRLS procedure.

#### 5-2 Zero padding effect on IRLS Procedure with Noise effect

Figure (9) The normalized magnitude output of the IRLS procedure. when it is applied to 25-bit to five different polyphase codes, zero-padded=100.

#### 5-3 Zero-padded effect on IRLS Procedure with Noise effect:

To introduce noise in the assessment of codes, the initial state of the simulator noise generator is unified with all tests. This is necessary for the comparison of the codes under test.

Figure (10) shows the PSL-versus-P behaviors of the IRLS procedure when it is applied to five different polyphase codes with SNRi equal to 10dB. While, Figure (11) shows the ISL-versus-P behaviors of the IRLS procedure when it is applied to five different polyphase codes with SNRi equal to 10dB, Moreover, Figure (12) shows the SNRL-versus-P behaviors of the IRLS procedure when it is applied to five different polyphase codes with SNRi equal to 10dB.

Figures (13), (14), and (15) show the PSL-versus-SNRi behavior, ISL-versus-SNRi behavior, and SNRL-versus-SNRi behavior of the IRLS procedure with filter length = 200 when it is applied to five different polyphase codes with different SNRi.



Figure (10): PSL-versus-P behavior of the IRLS procedure when it is applied to five different polyphase codes with SNRi=10dB.



Figure (11): ISL-versus-P behavior of the IRLS procedure when it is applied to five different polyphase codes with SNRi=10dB.



Figure (12): SNRL-versus-P behavior of the IRLS procedure when it is applied to five different polyphase codes with SNRi=10dB.



Figure (13): PSL-versus-SNRi behavior of the IRLS procedure with filter length = 200 and when it is applied to five different polyphase codes with different SNRi.



Figure (14): ISL-versus-SNRi behavior of the IRLS procedure with filter length = 200 and when it is applied to five different polyphase codes with different SNRi.



Figure (15): SNRL-versus-SNRi behavior of the IRLS procedure with filter length = 200 and when it is applied to five different polyphase codes with different SNRi.

# 6. Conclusions

1. In Noise Free case, the (Frank, P1, P2, P3 and P4) polyphase codes and IRLS procedure is used to observe the relationship between filter length (P) and The Peak Sidelobe Level (PSL), the Integrated Sidelobe Level (ISL) and the Signal-to-Noise Ratio Loss (SNRL) criteria. 2. In Figures(6,7), the (PSL and ISL) curves for both (P1 and P2) polyphase codes are perfect match for each other, the (PSL and ISL) curves for both (P1 and P2) polyphase codes are match too and the (PSL and ISL) curves for Frank polyphase code is the best among the other codes.

3.In Figure (8), the (SNRL) curves for both (P1 and P2) polyphase codes are match replica, as well as the relation between (P3 and P4) polyphase codes are match too and it is found that, the Frank code is the best code in term of SNRL (i.e. least losses) for the range (P=N till P=N+120); while the range from (P=N+140 till P=N+200), both (P3 and P4) are outperform the other codes.

4. In Additive White Gaussian Noise case, it is found that, the P3 code is the best code in terms of (PSL and ISL) criteria within an acceptable (SNRL).

5. In IRLS procedure, for odd code the filter length value must be even and vise versa (i.e. for even code the filter length value must be odd).

6. The length of IRLS procedure is increasable and the performance of IRLS procedure is improved too. But when the filter length is increased the (PSL and ISL) are increased too, but this will increase in the losses.

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