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Non Linear Finite Element Analysis of Confined HSC Columns Under Concentric and Eccentric Loadings.

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Abstract

The high compressive strength concrete is especially advantageous in compressed members such as columns, which can be made more slender and consequently, make economic benefits possible. This paper presents a nonlinear finite element analysis of eleven square high – strength plain and reinforced concrete columns confined with transverse steel under concentric and eccentric compressive loading. In this study, the columns were modeled as discrete elements using ANSYS 12.1 finite element software. Concrete was modeled with 8-node SOLID65 elements that can translate either in the x-, y-, or z- axis direction. Longitudinal and transverse steels were modeled as discrete elements using 3D-LINK 8 bar elements. The nonlinear constitutive laws of unconfined and confined reinforced concrete columns were implemented in the modeling. The results indicate that the axial stress-strain relationships obtained from the analytical model using ANSYS are in good agreement with the experimental data. This has been confirmed with the insignificant difference (1% to 10%) between the analytical and experimental results. The comparison shows that the ANSYS nonlinear finite element program is capable of modeling and predicting the actual nonlinear behavior of unconfined and confined high – strength concrete column under concentric and eccentric loading. The axial stress – strain relationship, the maximum applied load have also been confirmed to be satisfactorily.

الخلاصة:

الخرسانة ذات مقاومة الانضغاط العالية يستفاد منها بشكل خاص لصناعة العناصر الانشائية المعرضة لقوى انضغاط عالية وخير مثال على ذلك الاعمدة الخرسانية. حيث يمكن الاستفادة من هذة الخاصية لصناعة اعمدة خرسانية اكثر نحافة واكثر اقتصادية. تقدم هذه الدراسة بحثا نظريا لتقصي السلوك اللاخطي للأعمدة الخرسانية المقيدة وغير المقيدة ذات مقاومة الانضغاط العالية باستخدام طريقة العناصر المحددة تحت تأثير الاحمال المحورية وغير المحورية.
لتقصي سلوك الأعمدة المقيدة وغير المقيدة، تم استخدام برنامج العناصر المحددة (ANSYS 12.1) لإنشاء نماذج رياضية معتمدة على العناصر المحددة ثلاثية الأبعاد واللاخطية لتمثيل الأجزاء الخرسانية وعناصر مستقلة (Discrete Elements) لتمثيل قضبان حديد التسليح. وبعد تحليل أحد عشر من الأعمدة الخرسانية المربعة المقيدة وغير المقيدة ذات مقاومة انضغاط عالية وتحت تأثير الأحمال المحورية وغير المحورية. أظهرت طريقة العناصر المحددة في التحليل

حصول توافق جيد بعد مقارنتها بالنتائج العملية حيث حصل تأكيد ذلك من خلال الفروقات القليلة (1% الى 10%) بين نتائج التحليل والنتائج العملية. ان هذه الطريقة جيدة لبيان السلوك اللاخطي للأعمدة المقيدة ذات مقاومة الانضغاط العالية حيث حصل تأكيد ذلك من خلال العلاقة بين الاجهاد والانفعال المحوريين والحمل الأقصى المسلط.

1. Introduction

One of several reasons that cause the collapse of a multi- story building or bridges structure is the failure of supporting members which cannot withstand several types of loading. The failure of these members is mostly due to the lack of shear- resisting capacity and insufficient ductility provided by little amount of transverse steel. It is well known that the ductility of a reinforced concrete column plays a very important role in preventing such failure. One of the effective way to improve the ductility of columns is by introducing sufficient transverse steel as confining steel for concrete core in columns. This effort is primarily intended to delay the sudden collapse of a column and force in further to fail in a ductile manner. The effectiveness of confinement depends on the uniformity of stress occurred around the perimeter interface between the confining steel and concrete core. In rectilinear column [(1)] section, the most effective confined regions are only located at the four corners of the confining steel as shown in figure (1). Figure (1) also shows the effectively confined concrete which is assumed arched between the points where the lateral steel exerts a confining pressure on the concrete. In the case of rectilinear lateral reinforcement, the area of effectively confined core is less than the core area even at the tie level and is further reduced away from the tie level. In the case of circular or spiral ties, the reduction of the core area to an effectively confined area takes place only along the longitudinal axis of the column.

The effect of confinement [(1)] in a reinforced concrete column can be considerably increased if:

1- The spacing of transverse steel is denser.

2- More number and well distributed longitudinal steel is used around the perimeter of the column section.

3- More number and well- distributed crossties are provided in the concrete core.

High- strength concrete (HSC) with compressive strength exceeding 50 Mpa has been increasingly used in various parts of the structure and specially columns.

As a result, many studies [(2-6)] have been conducted on HSC members. One of main objectives of such studies is to investigate the validity of design codes, which are primarily empirical and are developed based on experimental studies on normal strength concrete (NSC) to include HSC. In these studies also different types of stress – strain relationships for concrete have been proposed for the nonlinear analysis of member behavior and for the ultimate state analysis of high – strength concrete elements under combined flexural and axial load.

The increase of strength and ductility of normal strength concrete columns afforded by well detailed lateral confinement reinforcement is well documented [(7-9)]. These studies carried though experimental tests on high- strength concrete columns and proposed a stress – strain model of ductile confined concrete.

Different methods have been utilized to study the response of structural components. Experimental testes have been widely used as means to analyze individual elements in order to achieve an acceptable accuracy for practical usage. While this method that produces real life response is extremely time consuming, and the use of materials can be quite costly. Besides it still depends on the availability and accuracy of the test apparatus and instrument. The use of finite element analysis has increased due to the progress knowledge and capability of computer software and hardware. Finite element method becomes the choice method to analyze concrete structural components, since it is much faster and extremely cost- effective.

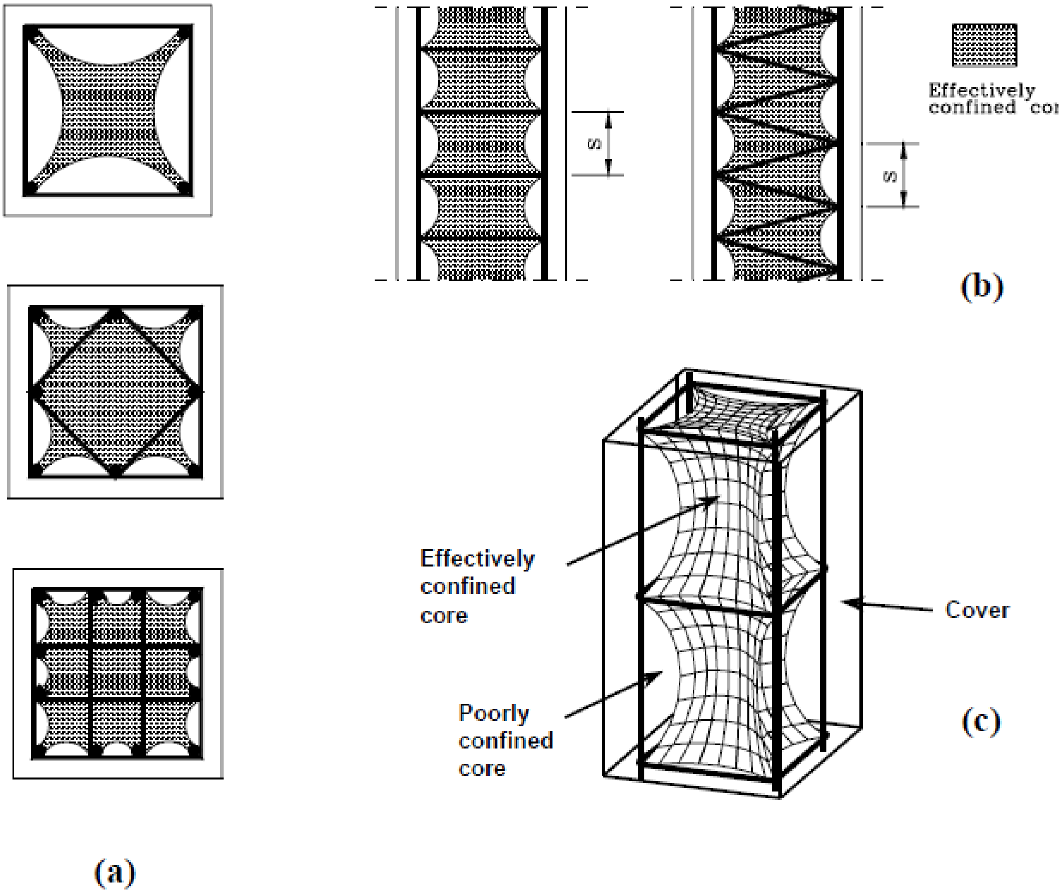


Figure (1): Effectively Confined area in tied Concrete Columns; a) Square b) Circular sections; c) 3D view of a Square Column[(1)].

In this study, the authors propose to use ANSYS12.1 [(10)], which is capable of modeling the nonlinear behavior of reinforced concrete beams [(11)]. Experimental tested specimens [2] used for predicting the nonlinear behavior of both unconfined and well confined square columns with various compressive strength ranging from 48 to 101 MPa, under concentric and eccentric compression load is selected to compare with analytical results.

2. Finite Element Modeling

The nonlinear finite element method has become popular over the last three decades for analyzing reinforced concrete members. This method yields a wide range of useful information from a single computer program. Such information includes displacements, strains, distribution of normal and shear stresses in concrete, crack pattern at different stages of loading and forces in longitudinal and transverse steel.

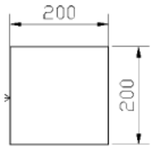
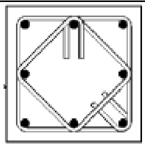
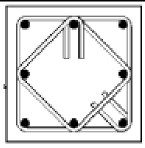
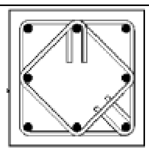
The ANSYS 12.1 computer program is utilized for analyzing all modeled columns throughout the current study.

2-1 Geometry Modeling

In this study, eleven columns were analyzed by ANSYS [10] programs. The analytical models were modeled according to the experimental column specimens [2] as shown in Table (1). The columns had a typical cross section of 200 X 200 mm with the height of 800 mm as listed in Table (1). The concrete cover was 15 mm. The first and second columns specimen namely S40- A-E0 and S70 –A-E0 was made from plain concrete.

The remaining columns (3 to 11) specimens were reinforced with 8ϕ 10 mm longitudinal bars and lateral reinforcement ties, ϕ 6 each 50 mm spacing. The two ends of all specimens, including the plain concrete ones, were reinforced and welded to 20 mm- thick plates. Each steel plate facilitates the fixing of specimen to the loading ring.

The specimens were tested under different loading schemes: concentric compression and eccentric compression with a constant eccentricity. The main variables are concrete strength, ranging from 48 to 101 MPa and confinement (lateral) reinforcement.

Specimen	f_c' (MPa)	Size (mm)	Eccentricity (mm)	Reinforcement	Section
S40 – A-E0	48.3	200x200x800	E = 0	Plain	
S70- A-E0	71.4	200x200x800			
S40- B- E0	48.3	200x200x800	E = 0	Long. Reinf. = 8ϕ 10 10 mm nominal diameter Ties = ϕ 6 @50 mm Ties clear cover =15 mm	
S70-B-E0	73.2	200x200x800			
S90-B-E0	100.6	200x200x800			
S40-B-E20	49	200x200x800	E =20	Long. Reinf.= 8ϕ 10 10 mm nominal diameter Ties = ϕ 6 @50 mm Ties clear cover =15 mm	
S40-B-E40	49	200x200x800	E = 40		
S40-B-E60	49	200x200x800	E = 60		
S70-B-E20	76.1	200x200x800	E = 20	Long. Reinf.= 8ϕ 10 10 mm nominal diameter Ties = ϕ 6 @!50 mm Ties clear cover =15 mm	
S70-B-E40	76.1	200x200x800	E = 40		
S70-B-E60	76.1	200x200x800	E =60		

2-2 ANSYS Finite Element Model

The FEA study includes the modeling of high-strength reinforced concrete column, with the dimensions and properties corresponding to the actual experimental data [2]. To create finite element model in ANSYS [10] there are multiple tasks that have to be completed for the model to run properly. The element type and material properties can be elaborated in the following details to reflect the actual mechanical and physical properties of the column specimens.

2-2-1 Element Types

2-2-1-1 *Reinforced Concrete*

Eight-node solid element, Solid65, was used to model the concrete. The solid element has eight nodes with three degrees of freedom at each node – translations in the nodal x, y, and z directions. The element is capable of predicting plastic deformation, cracking in three orthogonal directions, and crushing. The geometry and node locations for this element type are shown in figure (2).

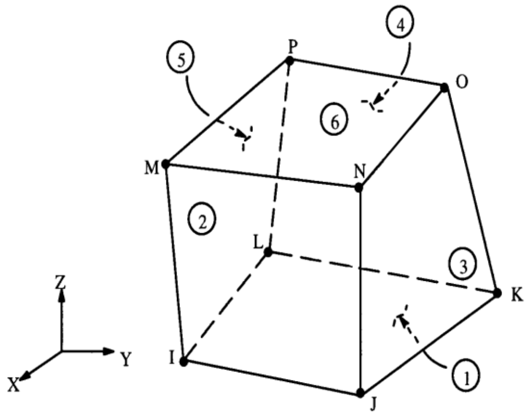


Figure (2): Solid65 -3-D reinforced concrete solid

Link8 element was used to model the steel reinforcement. Two nodes are required for this element. Each node has three degrees of freedom, – translations in the nodal x, y, and z directions. The element is also capable of plastic deformation. The geometry and node locations for this element type are shown in figure (3).

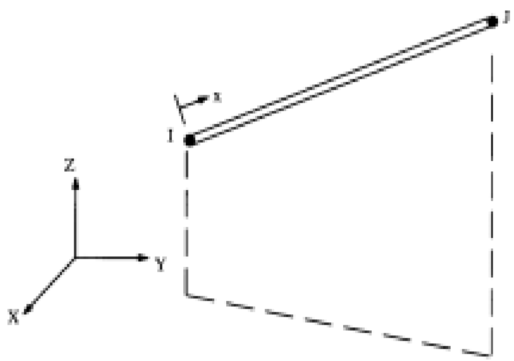


Figure (3):Solid45-3-D solid (ANSYS)

2-2-1-2 *Steel Plates*

Eight-node solid element, Solid45, was used to model the steel plates at the column supports. The element is defined with eight node having three degrees of freedom at each node – translations in the nodal x, y, and z directions. The geometry and node locations for this element type are shown in figure (4).

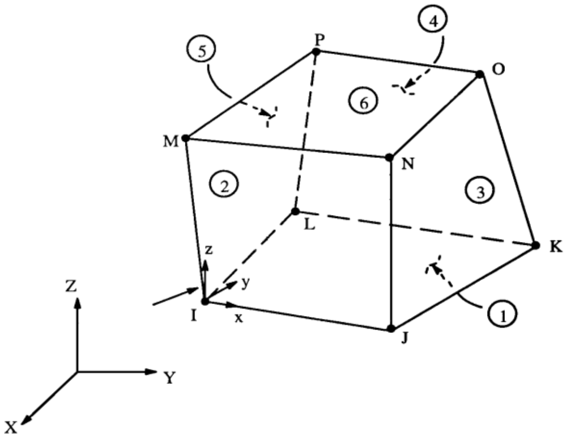


Figure (4): Solid45-3-D solid (ANSYS)

2-2-2 Material Properties

2-2-2-1 *Concrete*

Concrete is a brittle material and has different behavior in compression and tension. In compression, the stress – strain curve relationship for concrete is described by multi- linear isotropic curve, linear elastic up to about 40 percent (12) of the maximum compressive strength (f_c'). The stress–strain curve for each column model is constructed from six points connected by straight lines to represents the multi-linear isotropic stress -strain curve for the concrete (13) as shown in figure (5).In tension, the stress – strain curve for concrete was assumed to be linearly elastic up to the ultimate tensile strength (f_t = 0.1 f_c') (13).

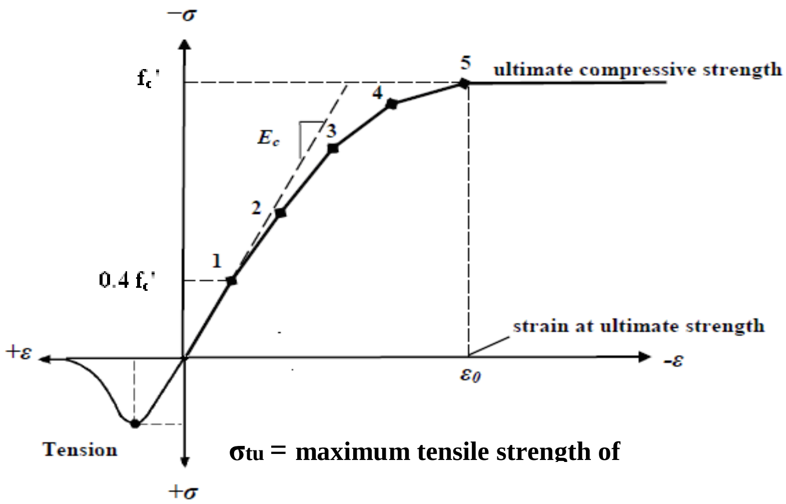


Figure (5): Simplified Uniaxial Compressive and Tensile Stress-Strain Curve for Concrete (13)

2-2-2-1-1 *FEM Input Data*

For concrete ANSYS requires input data for material properties as follows:

1- Elastic modulus ($E_c = 4700 \sqrt{fc'}$).

2- Ultimate uniaxial compressive strength (f_c').

3- Ultimate uniaxial tensile strength (f_t, assumed to be ft =0.1 f_c') [13].

4- Poisson's ratio (ν) assumed to be ν =0.2.

5- Shear transfer coefficient (β_0) for open cracks and (β_c) for closed cracks, representing conditions of crack face for determining the amount of shear transfer across the crack were used. The range from (0,0) to (1.0). In present study, (β_0) was assumed to be (0.2) while (β_c) was (0.6).

6- Compressive uniaxial stress – strain relationship for concrete.

7- Concrete density = 2400 kg/m^3.

A summary of the concrete properties used in the finite element modeling study is shown in Table (2).

The steel plates were added at top and bottom location for all columns in finite elements (as in the actual columns) to provide a more even stress distribution over the top and bottom areas. An elastic modulus equal to 200,000 MPa and Poisson's ratio of 0.3 were used for the plates. The steel plates were assumed to be linear isotropic elastic materials.

Table (2): Summary of Material Properties for Concrete

Specimen	f_c' MPa	f_t MPa	E_c Mpa	ν	β_0	β_c
S40-A-E0	48.3	4.83	32664	0.2	0.2	0.6
S40-B-E0	48.3	4.83	32664	0.2	0.2	0.6
S70-A-E0	71.4	7.14	39714	0.2	0.2	0.6
S70-B-E0	73.2	7.32	40212	0.2	0.2	0.6
S90-B-E0	100.6	10.6	47140	0.2	0.2	0.6
S40-B-E20	49	4.9	32900	0.2	0.2	0.6
S40-B-E40	49	4.9	32900	0.2	0.2	0.6
S40-B-E60	49	4.9	32900	0.2	0.2	0.6
S70-B-E20	76.61	7.61	41001	0.2	0.2	0.6
S70-B-E40	76.61	7.61	41001	0.2	0.2	0.6
S70-B-E60	76.61	7.61	41001	0.2	0.2	0.6

2-3 Element Meshing

After preparing all the input data of material and geometrical properties, the column models were divided into small cubical or rectangular elements, as shown in figure (6). The plain and reinforced concrete specimens elements connected at the top and bottom with steel plate while for column specimens reinforced with longitudinal and lateral (Ties) reinforcement, elements was created according to the location of reinforcing bars either the longitudinal or lateral reinforcement, as well as the column specimen cross – sectional perimeter. By using sharing nodes option in ANSYS, SOLID65 and Link8 elements can be interconnected one to another forming a single solid column model which is capable of simulating the actual behavior of reinforced concrete column.

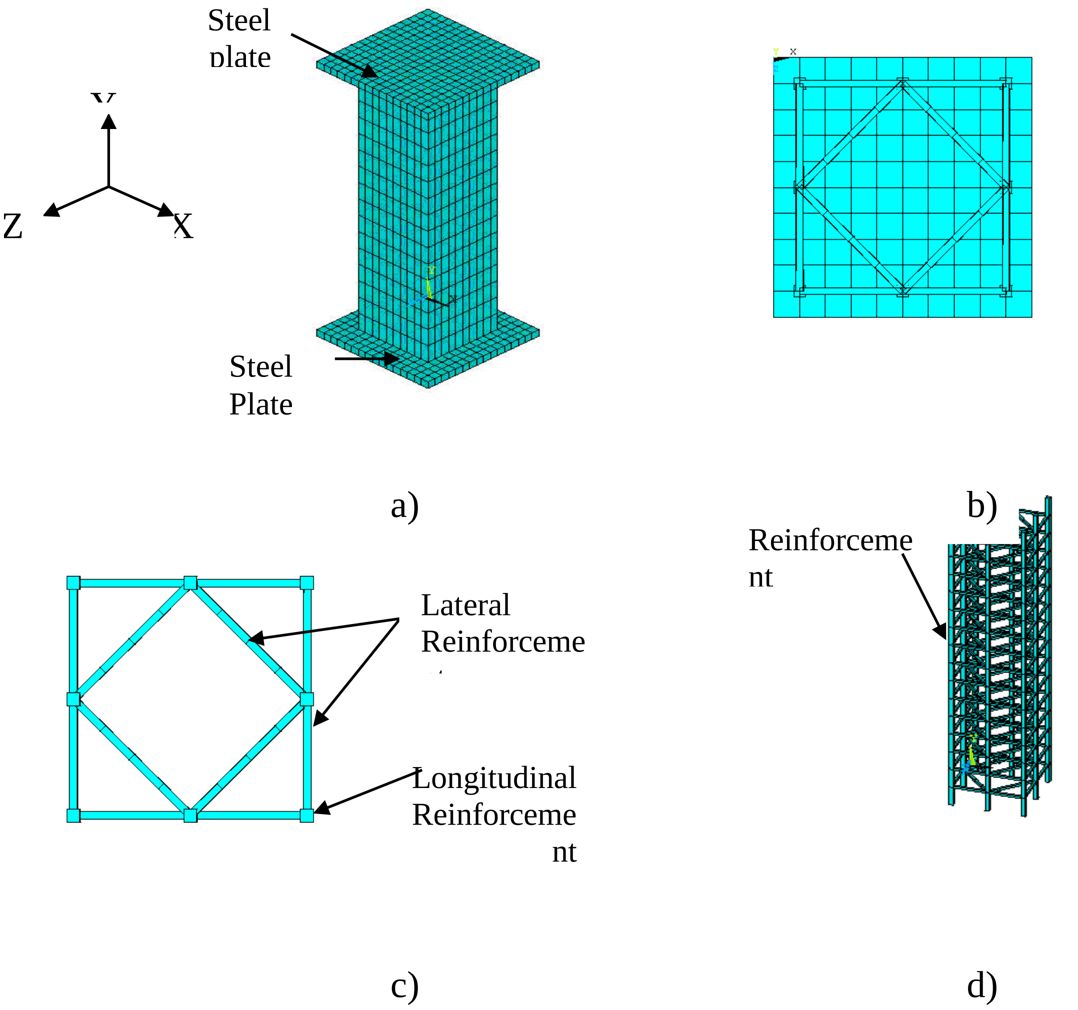


Figure (6): Mesh of Columns: a) Concrete Column 3D View, b) Cross Section in Column, c) Cross Section in Reinforcement, d) 3D view in Reinforcement.

2-4 Loads and Boundary conditions

Displacement boundary conditions are needed to be constrained in the model to get a unique solution. To ensure that the model acts the same way as the experimental columns specimens, boundary conditions need to be applied where the supports and loadings exist.
For concentric columns model the displacement of all nodes at bottom base of column in x, y and z. directions is held zero (Ux=0 , Uz=0) and Uy=0) , while for eccentric column model the displacement of nodes at centerline in z- direction of bottom base in x ,y and z – directions is held zero (Ux=0 ,Uz=0 and Uy=0) and the displacement of nodes at centerline in z-direction of top base in x and z-direction is held zero (Ux=0 and Uz=0).
To apply the axial load on the top of the concentric column specimens, an axial pressure was implemented over the entire top surface of the columns model, while the applied of eccentric load for eccentric columns model was simulated by an axial pressure implemented over entire the top and bottom surface plus two opposite couple force at extreme edge on both sides of column face in z- direction for both top and bottom base. The couple force was divided in to equal and opposite equivalent nodal forces.

3. Results and Discussions

In this study, it was found that if the crushing capability of the concrete is turned on, the finite element column models fail prematurely. Crushing of the concrete started to develop in elements located directly under the loads. Subsequently, adjacent concrete elements crushed within several load steps as well, significantly reducing the local stiffness. Finally, the model showed a large displacement, and the solution diverged. Therefore, in this study, the crushing capability was turned off. The crack patterns are not presented in the experimental work [(2)], so that it cannot make a comparison, therefore the cracking capability was also turned off. The final load applied from finite element analysis, is the last load before the solution diverged.
During this study a verification is carried out in order to check the validity and accuracy of the finite element procedure. The accuracy is determine by ensuring that axial stress-strains relation ship, axial stress distribution and maximum load is reasonably predicted compared with experimental results [(2)].

3-1 Stress –Strain Relationship

The axial stress-strain curves obtained from ANSYS solution (δ_y and ε_y) are compared with experimental results [(2)], as shown in figure (7) and figure (8), its shows that the predictions are in close agreement with experimental curves. This indicated that the actual behavior of unconfined and confined column specimens with transverse steel under concentric and eccentric compressive loading can be accurately predicted by the FEM approach.The accuracy of the proposed produce is also confirmed through the close value of compressive stress and compressive strain at maximum load, which is the final load from the finite element models of the last applied load before the solution diverged, compared with experimental results [2] as shown in Table (3) and Table (4). These values showed the accuracy of the proposed procedure in predicting the actual nonlinear behavior of columns under concentric and eccentric loading, especially for plain concrete.

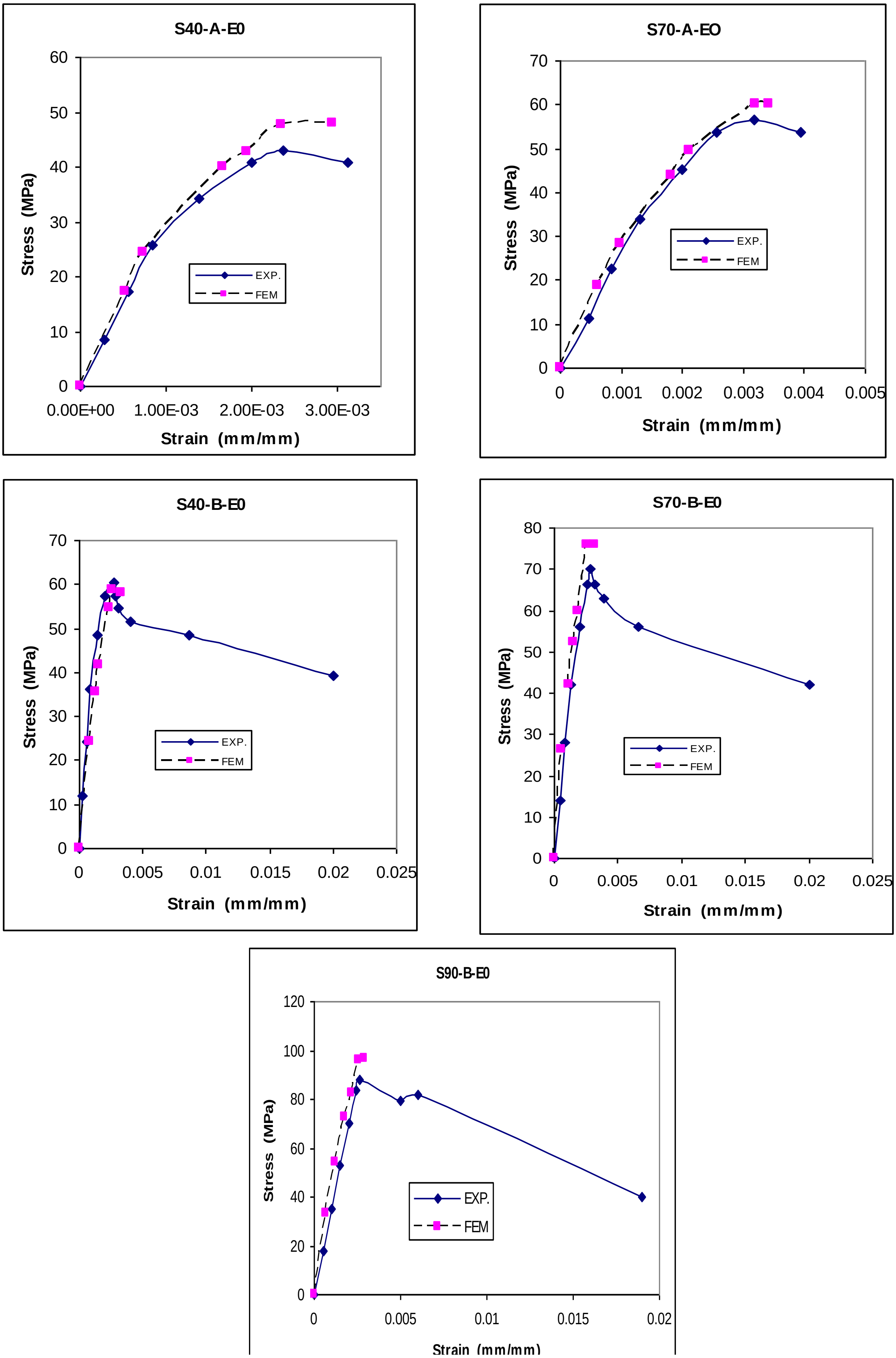
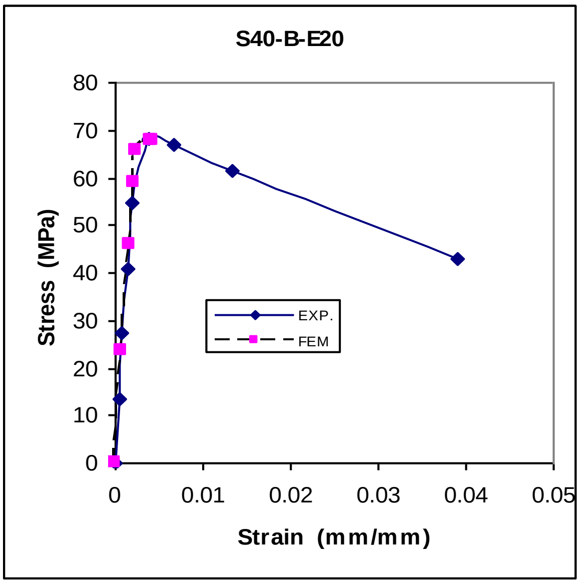
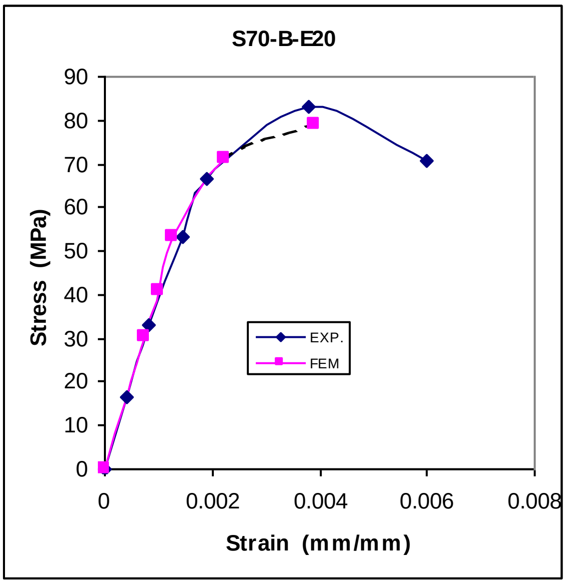
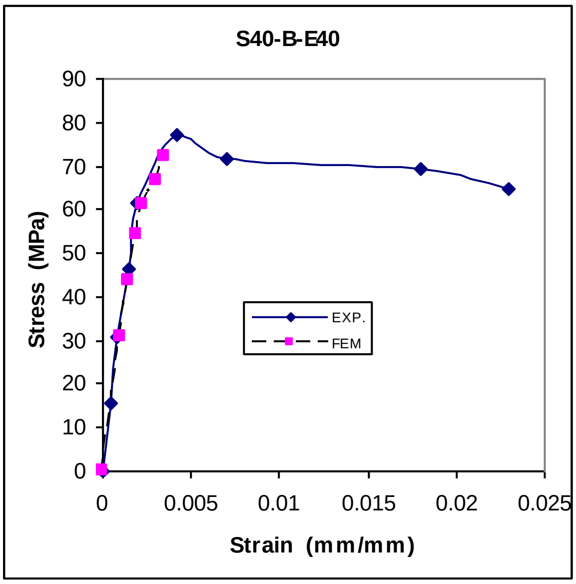
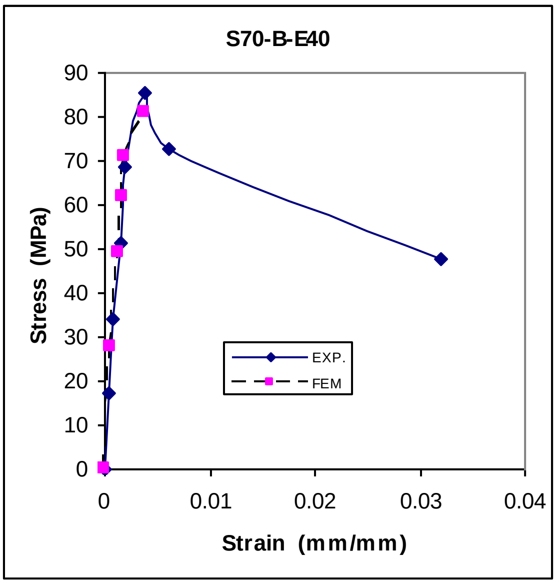


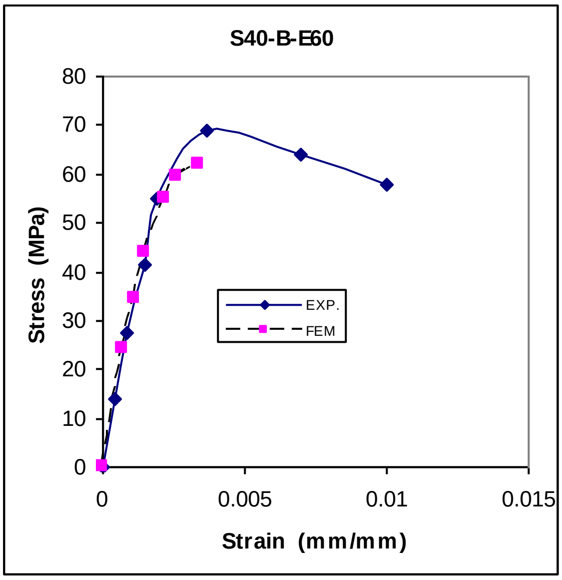
Figure (7): Axial Stress- Strain Relationship for Concentric Specimens.


S40-B-E20
Stress (MPa)
0
10
20
30
40
50
60
70
80
0
0.01
0.02
0.03
0.04
0.05
Strain (mm/mm)
EXP.
FEM


S70-B-E20
Stress (MPa)
0
10
20
30
40
50
60
70
80
90
0
0.002
0.004
0.006
0.008
Strain (mm/mm)
EXP.
FEM


S40-B-E40
Stress (MPa)
0
10
20
30
40
50
60
70
80
90
0
0.005
0.01
0.015
0.02
0.025
Strain (mm/mm)
EXP.
FEM


S70-B-E40
Stress (MPa)
0
10
20
30
40
50
60
70
80
90
0
0.01
0.02
0.03
0.04
Strain (mm/mm)
EXP.
FEM


S40-B-E60
Stress (MPa)
0
10
20
30
40
50
60
70
80
0
0.005
0.01
0.015
Strain (mm/mm)
EXP.
FEM

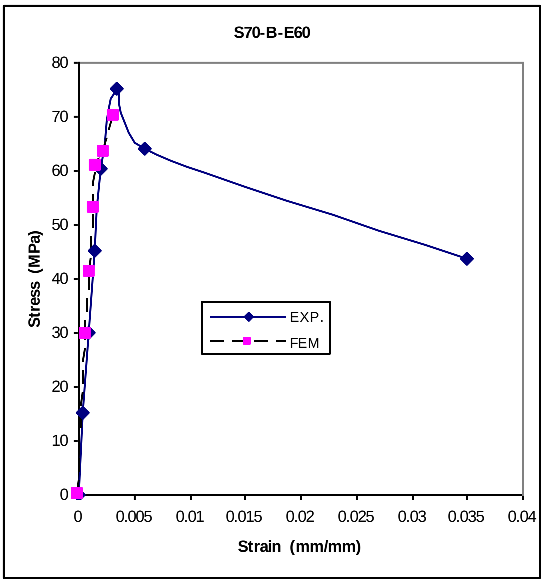

S70-B-E60
Stress (MPa)
0
10
20
30
40
50
60
70
80
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
Strain (mm/mm)
EXP.
FEM

Table (3): Comparison between Experimental [2] and FEA loads, Stress and Strain for Concentric Specimens.

Specimens	Experimental			FEA		
	Max.load kN	Compressive stress, MPa	Compressive strain, mm/mm	Max.load kN	Compressive stress, MPa	Compressive strain, mm/mm
S40-A-E0	1716.4	42.91	0.00236	1802.22	46.8	0.002341
S70-A-E0	2256.6	56.415	0.00318	2369.43	60	0.00343
S40-B-E0	2421.8	60.545	0.00273	2300.71	58.6	0.00259
S70-B-E0	2799.0	69.975	0.00283	2938.95	75.8	0.00255
S90-B-E0	3523.6	88.075	0.00266	3699.78	96.6	0.00289

FEA/Expr.			
Specimens	Max.load kN	Compressive stress, MPa	Compressive strain, mm/mm
S40-A-E0	1.05	1.09	0.99
S70-A-E0	1.05	1.063	1.078
S40-B-E0	0.95	0.967	0.948
S70-B-E0	1.05	1.083	0.9
S90-B-E0	1.05	1.096	1.086

Table (4): Comparison between Experimental [2] and FEA loads, Stress and Strain for Eccentric Specimens.

Specimens	Experimental			FEA		
	Max.load kN	Compressive stress, MPa	Compressive strain, mm/mm	Max. load kN	Compressive Stress, MPa	Compressive Strain, mm/mm
S40-B-E20	1708.5	68.34	0.00387	1623	67.2	0.00393
S40-B-E40	1392	76.99	0.00391	1322.4	72.2	0.00353
S40-B-E60	984.6	68.9	0.00369	886.14	62	0.00336
S70-B-E20	2074.6	82.98	0.00379	2178.33	78.8	0.00392
S70-B-E40	1556.6	85.6	0.00373	1478.74	80.9	0.00375
S70-B-E60	1075.3	75.27	0.00348	1021.535	70	0.0033

FEA/Expr.			
Specimens	Max. load kN	Compressive Stress, MPa	Compressive Strain, mm/mm
S40-B-E20	0.95	0.98	1.015
S40-B-E40	0.95	0.937	0.902
S40-B-E60	0.9	0.9	0.91
S70-B-E20	1.05	0.949	1.007
S70-B-E40	0.95	0.945	1.005
S70-B-E60	0.95	0.929	0.948

3-2 Stress Distribution

The axial stress distributions of column specimens S40-A-E0 and S70-A-E0 obtained from the ANSYS solution are shown in figure (9). Higher axial stress concentration occurs over the center region of the column cross section, since these two specimens does not contain any reinforcing bars (plain concrete), the stress contour describes a correct mechanism [14] of a plain concrete column specimen subjected to concentric loading.

The axial stress contours over mid-height cross sections of the concentric specimens S40-B-E0, S70-B-E0 and S90-B-E0 are show in figure (10), indicate similar axial stress distributions with various intensities of stress concentrations. The axial stress concentrations around the longitudinal reinforcement also indicate similar axial stress distributions with the axial stress distribution in the actual column.

The axial stress contours of the eccentric column specimens S40-B-E0, S40-B-E40, S40-B-E60, S70-B-E20, S70-B-E40 and S70-B-E60 are shown in figure (11). This figure indicates higher compressive stress intensities in the loaded side at small eccentricities (20 & 40mm) and higher tensile stress concentrations over the least part at unloaded side of the column section at large eccentricities (60mm), which is describes a correct mechanism[14] of a column specimen subjected to eccentric compressive load with constant eccentricities.

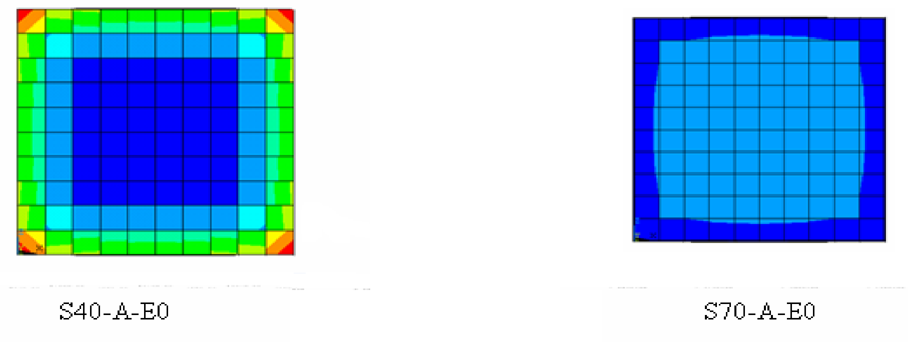


Figure (9): Axial Stress Contours of Unconfined Concentric Columns at Mid Height.

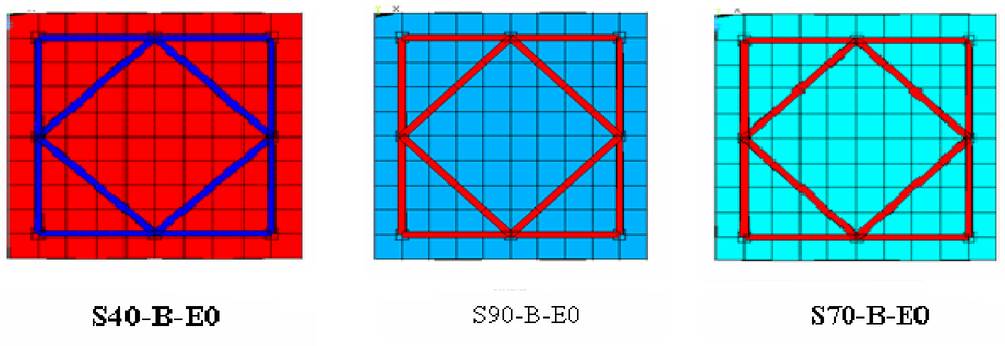
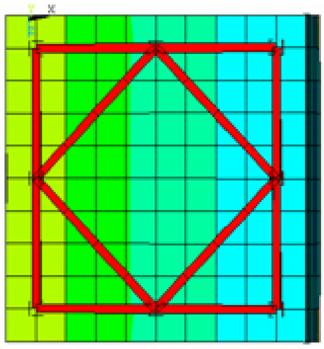
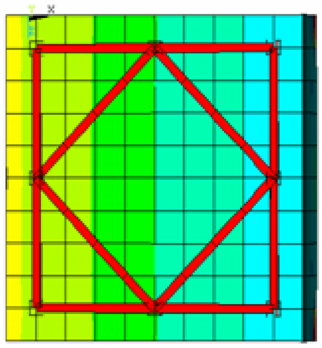


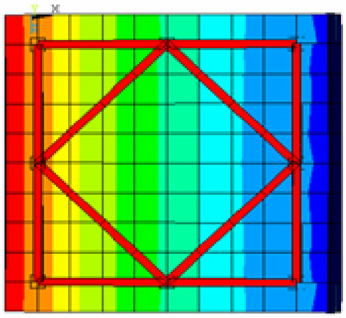
Figure (10): Axial Stress Contours of Confined Concentric Columns at Mid Height.



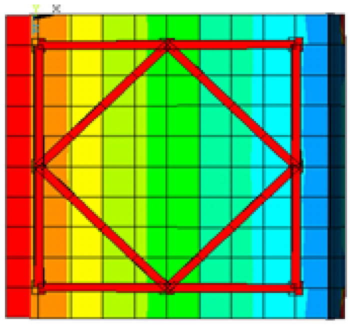
S40-B-E20



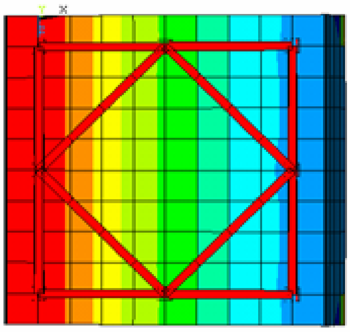
S70-B-E20



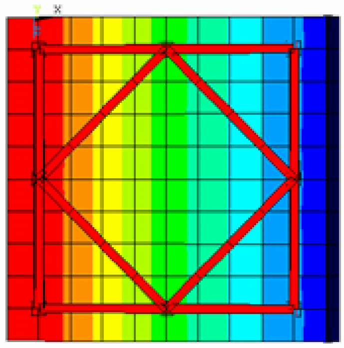
S40-B-E40



S70-B-E40



S40-B-E60



S70-B-E60

Figure (11): Axial Stress Contours of Confined Eccentric Columns at Mid Height.

4. Conclusions

Based on finite element analysis and discussion above, the following conclusions can be drawn:

1. ANSYS software is capable of predicting the actual stress- strain relationships of both unconfined and confined reinforced concrete column specimens subjected to concentric and eccentric loading.
2. The accuracy of the proposed procedure has been well confirmed by the close value of maximum compressive load, compressive stress and compressive strain obtained from the FEM analysis which is raging from 1% to 10% compared with the experimental results.
3. From the axial stress contours obtained from the FEM analysis for concentric specimens, it can be concluded that the axial stress concentrations are in the center regions of the column cross sections for plain concrete specimens, while the axial stress concentrations around the longitudinal reinforcement, practically in the confined areas for reinforced concrete specimens.
4. From the axial stress contours obtained from the FEM analysis for eccentric specimens, it can be concluded that the higher compressive stress intensities in the loaded side at small eccentricities (20 & 40mm), while higher tensile stress concentrations over the least part at unloaded side of the column section at large eccentricities (60mm).

5. References

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