Sidelobe-Suppression Technique Applied To Binary Phase Barker Codes

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Abstract

This paper presents binary phase Barker codes and corresponding decoding filters. These filters are optimal in the sense that they produce no sidelobes and they maximize the signal-to noise ratio. Here, the input code is sampled at a rate of N_s.

Key Words: binary phase Barker codes, matched filter, mismatched filter (sidelobe-free decoding filter).

الخلاصة

يقدم هذا البحث شفرات باركر ثنائية الأطوار و مرشحات حل الشفرة المطابقة. هذه المرشحات مثالية بمعنى أنها لا تنتج قمم جانبية و هي تزيد نسبة الإشارة إلى الضوضاء. هنا, شفرة الإدخال مأخوذة العينة بمعدل N_S.

Introduction

Phase modulation is a principle, which divides the radar pulse into a set of sub-pulses of equal duration, and the phase of each sub-pulse is fixed. When two phase values with a phase difference of 180° is used, the modulation is a binary code.

Biphase modulation of a radar transmission is a well-known method for increasing radar transmission power, while still maintaining a good range resolution.

Matched filter is a filter, which creates unwanted sidelobes at the output of the receiver. These sidelobes can be eliminated by using a mismatched filter. However, there is an associated loss in signal-to-noise ratio (SNR).

In [1], Damtie, B, et. al., presented long optimal binary phase code-mismatched filter pairs that may be used in several applications including ionospheric radar measurements. This was done by investigating $1.04e^{09}$ number of binary phase codes. Rohling and Plagg in 1989 have published a deferent approach of eliminating the sidelobes in periodical binary phase codes by using mismatched filter [2]. Exhaustive search for optimal aperiodic binary phase codes and mismatched filter pairs up to length of 25 has been carried out (by Lehtinen, et al., in 2004) [3].

Coding Filter: [3]

Assume that a code consisting of N pulses (an N-bit code) such that the pulse length T_p is a multiple of the sampling interval T, i.e., $T_P=N_sT$, where N_s is an integer indicating the number of samples per bit. This means that the possibility of over-sampling is taken into account. Thus, by choosing T as the time unit, an elementary pulse can be written as:

$$P(n)=\sum_{i=0}^{Ns-1}\delta(j-n) \qquad , n=-\infty,\ldots,\infty \qquad (1)$$

where δ is the discrete time-impulse (unit sample; not to be confused with the delta function) given as:

$$\delta(n)=\begin{cases}1 & when; n=0\\ 0 & when; n\neq 0\end{cases} \qquad (2)$$

Accordingly, the impulse response of a coding filter of an N-bit binary code can be written as:

$$h_c(n)=\sum_{j=0}^{N-1}a(j)\delta(n-jN_s) \qquad , n=-\infty,\ldots,\infty \qquad (3)$$

Where a(j) = ± 1 when j=0, 1, . . ., N-l. The sequence of numbers a(j) defines the binary code. Note that $h_c(n)$ is zero when n < 0 or n > N_s (N-l).

General Transmission Code: [3, 4]

A code with length L can be described as an infinite length sequence with a finite number of nonzero pulses with phases and amplitudes defined by parameters Φ_k and a_k. These parameters obtain values $\Phi_k \in [0,2\pi]$ and $a_k[a_{min},a_{max}]$, where $k\in[1,\ldots\ldots,L]$. In the case of binary phase codes, the number of phases has been restricted $\Phi_k \in [0,\pi]$ and in most traditional work, the amplitudes have been set to 1.

The transmission code is obtained by means of a convolution as:

$$\varepsilon(n)=h_c(n)*p(n)$$

$$=\sum_{j=-\infty}^{\infty}p(j)h_c(n-j) \qquad , n=-\infty,\ldots,\infty \qquad (4)$$

where * denotes the convolution. Note that ε(n) is zero when n < 0 or n >N_S(N-1)

Standard Matched Filter: [3]

The impulse response of the standard matched filter is a mirror image of the code:

$$\mu(n)=\varepsilon(-n)=h_c(-n)*p(n) \qquad , n=-\infty,\ldots,\infty \qquad (5)$$

The output of the standard matched filter, or weight function, is given as:

$$w_m(n) = \mu(n) * \varepsilon(n) = h_c(-n) * p(n) * h_c(n) * p(n) \quad , n=-\infty,\ldots,\infty \qquad (6)$$

Sidelobe-Free Decoding Filter [3]

For designing a sidelobe-free decoding filter, first the impulse response can be defined as:

$$h_d(n) = \sum_{j=-\infty}^{\infty} b(j)\delta(n - jN_s) \qquad (7)$$

where the sequence of real numbers b(j) will be chosen to decode $h_c(n)$ in Eq. (3). In addition, an impulse response q(n) is needed for filtering the elementary pulse p(n). Thus, the complete structure of the sidelobe-free decoding filter for processing the echoes is given by:

$$\lambda(n) = h_d(n) * q(n)$$
$$= \sum_{j=-\infty}^{\infty} q(j) h_d(n - j) \quad , n=-\infty,\ldots,\infty \qquad (8)$$

The decoding of a binary phase coded signal can be carried out by means of a decoding filter such that the convolution of the decoding filter $h_d(n)$, the filter matched to the elementary pulse q(n) and the code ε(n) is a function with a desired shape. This shape defines the range resolution. Mathematically, this means that:

$$\lambda(n) * \varepsilon(n) = w(n) \qquad (9)$$

The result of the convolution w(n) is a weight function, or the output of the mismatched filter, which determines the range resolution and the range ambiguity functions. Fourier transforms of convolutions are products of the Fourier transforms of the convoluted sequences and thus the Fourier transform of the weight function w(n) is given by:

$$F\{w(n)\} = F\{h_d(n)\} F\{q(n)\} F\{\varepsilon(n)\}$$
$$= F\{h_d(n)\} F[\{h_c(n))\} \times F\{q(n)\} F\{p(n)\}] \qquad (10)$$

If $h_d(n)$ is chosen to make $F\{h_d(n)\}$ $F\{h_c(n)\} = 1$, then:

$$h_d(n) = F^{-1}\{\frac{1}{F\{h_c(n)\}}\} \qquad (11)$$

The inverse Fourier transform of Eq. (10) gives:

$$w = q(n) * p(n) \qquad (12)$$

Thus, the impulse response defined by Eq. (11) makes a sidelobe-free decoding filter producing exactly the same weight function to what would result from using no coding at all,

just the elementary pulse p(n) and a filter q(n) matched to it. In particular, no sidelobes are produced.
By combining Eqs. (8), (10), (12) in a proper manner, the mathematical expression for the transfer function of the complete sidelobe-free decoding filter that gives w(n) with a desired shape can be easily obtained and it is given by:

$$\Lambda(w) = F\{\lambda(n)\} = \frac{Q(w)}{H_c(w)} \qquad (13)$$

Where

$$Q(w) = \sum_{n=-\infty}^{\infty} q(n)e^{-jnw} \qquad (14)$$

And

$$H(w) = \sum_{n=-\infty}^{\infty} q(n)e^{-jnw} \qquad (15)$$

Finally, the impulse response of the sidelobe-free decoding filter is obtained by means of the inverse Fourier transform, which is:

$$\lambda(n) = F\{\Lambda(w)\} = \frac{1}{2\pi}\int_{w=0}^{2\pi} \frac{Q(w)}{H(w)} e^{jnw} dw \qquad (16)$$

SNR Performance of a Decoding Filter: [3]

There is a decrease in SNR when one applies a sidelobe-free compression filter instead of the standard matched filter. In this section, the SNR performance of sidelobe-free decoding of different Barker codes is investigated by comparing it with that of the corresponding matched filter. If the power spectral density of white noise entering a filter with a transfer function H(v) is $S(v)=S_n$, the total output noise power is:

$$P_n = S_n |H(v)|^2 dv = S_n \int_{-\infty}^{\infty} h^2(t)dt \qquad (17)$$

Hence, the SNR given by the matched filter is:

$$SNR_m = \frac{P_{w_m^2}}{S_n \sum_{n=-\infty}^{\infty} \mu(n)^2} \qquad (18)$$

where w_m^2 is the peak value of the weight function $w_m(n)$ of the matched filter and P is a scaling coefficient defining the received power. In a similar manner, the SNR value at the output of the sidelobe-free decoding filter is:

$$SNR_s = \frac{P_{w_s^2}}{S_n \sum_{n=-\infty}^{\infty} \lambda(n)^2} \qquad (19)$$

where w_s^2 is the peak value of the weight function $w_s(n)$ of the sidelobe-free decoding filter. The noise performance of different sidelobe-free filters can be compared with that of the matched filter by calculating the ratio of the two signal-to-noise ratios. Since the sidelobe-free decoding filter is designed to give (this is illustrated in Figs. 3 and 4), this parameter is:

$$R = \frac{SNR_s}{SNR_m} = \frac{\sum_{n=-\infty}^{\infty} \mu(n)^2}{\sum_{n=-\infty}^{\infty} \lambda(n)^2} \qquad (20)$$

The values of $\mu(n)$ and $\lambda(n)$ needed in Eq. (20) are obtained from Eq. (5) and Eq.(16), respectively. A sufficient accuracy for comparison purposes is obtained by truncating λ at the points where its absolute values are below 10^{-3}.

Results

Since the data analysis is based on discrete samples, and the theory is presented in terms of discrete signals. This leads to results, which can be used in MATLAB programming language.

Table 1 gives the values of R for Barker codes of different lengths. They illustrate the fact that sidelobe-free decoding of Barker codes degrades the SNR by about (5%-30%) relative to standard decoding. However, the degradation is smallest for the 5-bit and 13-bit Barker codes which are often used in incoherent scatter radar measurements. In the case of the 13-bit code, the loss is only about 5%. Hence, in this respect the 13-bit code is the optimal Barker code. Figure (1) indicates that, the coefficients of the sidelobe-free compression filter, applied to the 13-bit Barker code sampled at a rate, are 10-based logarithm of the absolute value of the their normalized coefficients. While the Figures (2) and (3) indicate the output of the corresponding matched filter and mismatched filter, respectively.

Table (1): SNR of the sidelobe-free decoding filters relative to that of matched filters for Barker codes of different lengths.

Length in Bits	Binary Barker Code	R (N_s=3)
3	+1 +1 -1	0.7441
4	+1 +1 -1 +1	0.6786
5	+1+1+1 -1 +1	0.8658
7	+1 +1 +1 -1 -1 +1 -1	0.7048
11	+1 +1 +1 -1 -1 -1 +1 -1 -1 +1 -1	0.7105
13	+1 +1 +1 +1 +1 -1 -1 +1 +1 -1 +1 -1 +1	0.9520

It is well-known that, the performance of matched-filter decoding of Barker codes is better than decoding without sidelobes. In the case of the 7-bit Barker code, it is shown that, the SNR given by sidelobe-free decoding is nearly 30% worse than that of standard decoding, but for the 13-bit code sidelobe-free decoding is only about 5% worse.

The deterioration of SNR should be evaluated against the benefits gained in disposing of the sidelobes, which, even for the 13-bit code, contribute by 7.1% to the total signal power from a homogeneous target. A practical example is shown where sidelobes mask a weak signal when the standard matched filter is used in the analysis. An improvement is achieved when sidelobe-free filtering is carried out.

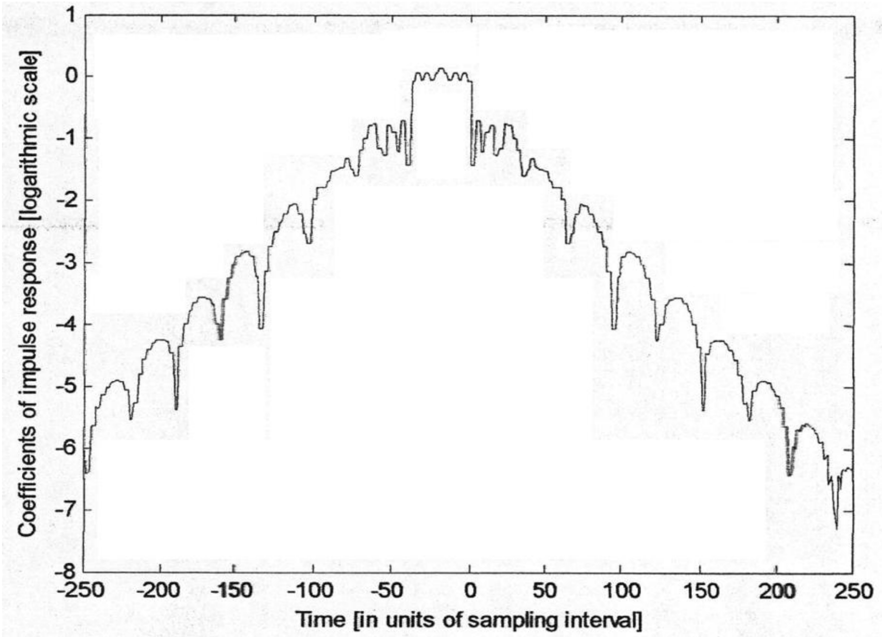

Figure (1): The 10-based logarithm of the absolute value of the coefficients of the sidelobe-free compression filter applied to the 13-bit Barker code sampled at a rate of 3 samples per bit.

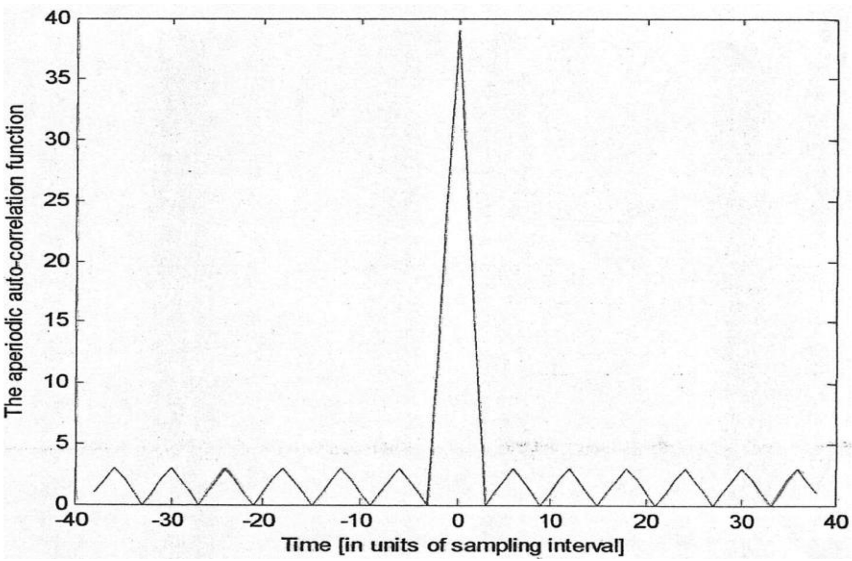

Figure (2): The matched filter output when the input is the 13-bit Barker code sampled at a rate of 3 samples per bit

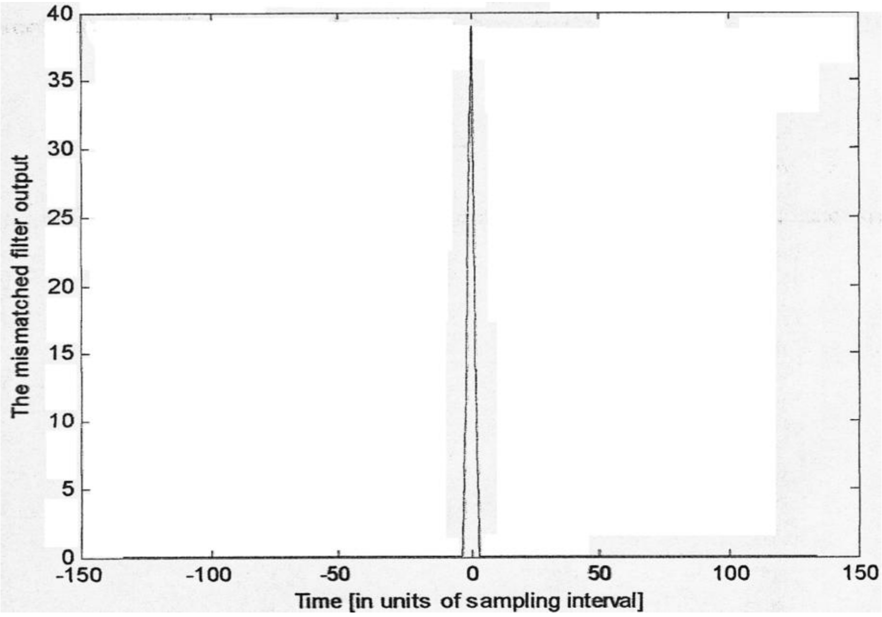

Figure (3): The weight function (The mismatched filter output) when the input is the 13-bit Barker code sampled at a rate of 3 samples per bit

References

[1]. Damtie, B., Lehtinen, M., Orispaa, M. and Vierinen, J., "Optimal long binary phase code-mismatched filter pairs with applications to ionospheric radars", Bull. Astr. Soc. India 35, pp. 619-623, 2007.

[2]. Rohling, H., and Plagge, W., "Mismatched-Filter Design for Periodic Binary Phased Signals", IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-25, No.6, pp. 890-897, November 1989.

[3]. Lehtinen, MS., Damtie, B., & Nygren T., "Optimal binary phase codes and sidelobe-free decoding filters with application to incoherent scatter radar", Annales Geophysicae, 22, pp. 1623-1632,8 April 2004.

[4]. Damtie, B., Lehtinen, M., Orispaa, M. and Vierinen, J., "General radar transmission codes that minimize measurement error of a static target", IEEE Transactions on Information Theory, Vol. 1, No. 11, November 2007.