Modified Softened Strut and Tie Model for Concrete Deep Beams

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Abstract

A modified softened strut-and-tie model (MSST) for determining the shear strength of reinforced concrete deep beams is proposed in this paper. It is a simplified analytical model and several improvements have been made. The method is based on Mohr Coulomb's failure criterion. The concrete softening effect and the stress distribution factor, k, based on force and moment equilibrium are satisfied. To validate the proposed method, the obtained results are compared with those available in the literature. The results indicate that the proposed method MSST is capable to predict the shear strength of variety of deep beams with acceptable accuracy.

الخلاصة

يقدم هذا البحث موديل رياضي معدل لطريقة الربط و الدعامة المضعفة MSST في حساب مقاومة القص للاعتاب الخرسانية المسلحة العميقة. تم الاعتمادعلى موديل مبسط للتحليل مع اجراء بعض التحسينات عليه. يستند هذا الموديل بالاساس على معيار الفشل المقترح من قبل مور و كولومب. استنادا" الى معادلات التوازن للقوى و العزم يتم تحقيق متطلبات كل من تاثير التضعيف للخرسانة و معامل توزيع الاجهادات، k. تم مقارنة النتائج المستحصلة من هذه الطريقة MSST مع النتائج المتوفرة في المصادر لاثبات مصداقية هذه الطريقة. بينت الارقام الناتجة من هذه الطريقة على دقة هذه الطريقة في تقدير مقاومة القص لانواع مختلفة من الاعتاب العميقة.

1. Introduction

Typically, reinforced concrete members are designed to resist shear and flexural forces based on the assumption that the strains vary linearly at a section. Referred to as the Bernoulli hypothesis or beam theory, the mechanical behavior of a beam is commonly determined by assuming that plane sections remain plane. The region of a structure where the Bernoulli hypothesis is valid is referred to as a B region. But, when the strains vary nonlinearly at a section, the Bernoulli hypothesis or beam theory cannot be used, the region is discontinuities (disturbed) and referred to as D region [1].

In general, the concrete deep beams are those having clear span do not exceed four times the overall member depth, or regions of beams with concentrated loads within twice the member depth from the support [2]. A deep beam design must be treated differently from a sectional design (or slender beam design) because the assumptions used to derive the sectional theory are not suitable for deep beams due to them not satisfying the plane section assumption. In practice, engineers commonly encountered deep beams when designing transfer girders, pile-supported foundations, shear walls, or corbels. However, the strut-and-tie model (STM) has been widely adopted in the analysis and design of reinforced concrete beams for about twenty years [3]. So far, the strut-and-tie model has been incorporated into American, Canadian and European standers. In the conventional STM (like in ACI code), the stresses are usually determined by the equilibrium condition alone, while the strain compatibility conditions are neglected. However, the Softening Strut-and-Tie model (SST) [4] has been proposed for determining the shear strengths of reinforced concrete deep beams, which satisfies equilibrium, compatibility and constitutive laws of cracked reinforced concrete.

The proposed Modified Softened Strut-and-Tie model MSST described in this paper is based on the failure criterion from the Mohr-Coulomb theory for nodal zones (tension-compression stress state). During the derivation, the factor k is determined from the consideration of both force and moment equilibrium. Based on the available experimental and theoretical data, the applicability of the proposed MSST model to deep beams for predicating the shear strength is examined. The results show that the proposed model is sufficiently accurate for the model predictions.

2. Modified Strut-and-Tie Model

2-1 Considering of Concrete Softening Effect

Cracked reinforced concrete in compression has been observed to exhibit lower strength and stiffness than uniaxially compressed concrete. The extent of the reduction in strength can be related to the value of the transverse tensile strain in the concrete. This phenomenon of strength and stiffness reduction is called the softening of concrete [5]. There are mainly three methods for determinate this phenomenon [3]:

i. According to statistical test results, concrete strength efficiency factors are adopted. Despite the vast amount of research done in this area [2,6], there is no clear consensus among research on the strength of struts and nodes [1]. So, the strength of struts and nodes depends mainly on the experience.

ii. Function expressions, the influence of principal strain are considered on the determining of compressive strength [7,8]. The method seems to be more accurate, but adds complexity because of the simultaneous application of equilibrium conditions, compatibility equations and stress-strain relationships.

iii. Linear interactive failure criteria, such as Mohr-Coulomb theory, are utilized to account for the softening effect directly.

The model utilizes a failure criterion from the modified Mohr-Coulomb theory for nodal zones (tension-compression stress state) as below:

$$\frac{f_1}{f_t}+\lambda\frac{f_2}{f'_c}=1 \qquad \ldots (1)$$

Where f_1 and f_2 are principal tensile and compressive stresses at the nodal zone respectively, f'_c is the concrete compressive strength of cylinder in the f_2 direction, f_t represents the maximum combined tensile strength of both reinforcement and concrete in f_1 direction, and λ is a factor represents the importance of tensile stress in the ultimate limit state. Kupfer and Gerstle [9] proposed $\lambda = 0.8$ for successful biaxial tension-compression criterion according to experimental results. In the present study, if the diagonal strut is reinforced by web reinforcement, $\lambda = 0.8$ is used and $\lambda = 1$ is taken for unreinforced diagonal strut.

2-2 Derivation of Shear Strength

2-2-1 Bottom Nodal Zone

From the equilibrium of forces at the bottom nodal zone of the inclined strut, the following equations can be obtained, as shown in Fig. (1):

$$F_c = \frac{V_n}{\sin\theta_s} \qquad \ldots (2)$$

$$T_s = \frac{V_n}{\tan\theta_s} \qquad \ldots (3)$$

where F_c and T_s are the forces in the primary strut and bottom tension tie, respectively. V_n is the shear strength of the beam. The inclined angle of the primary strut Θ_s can be computed from

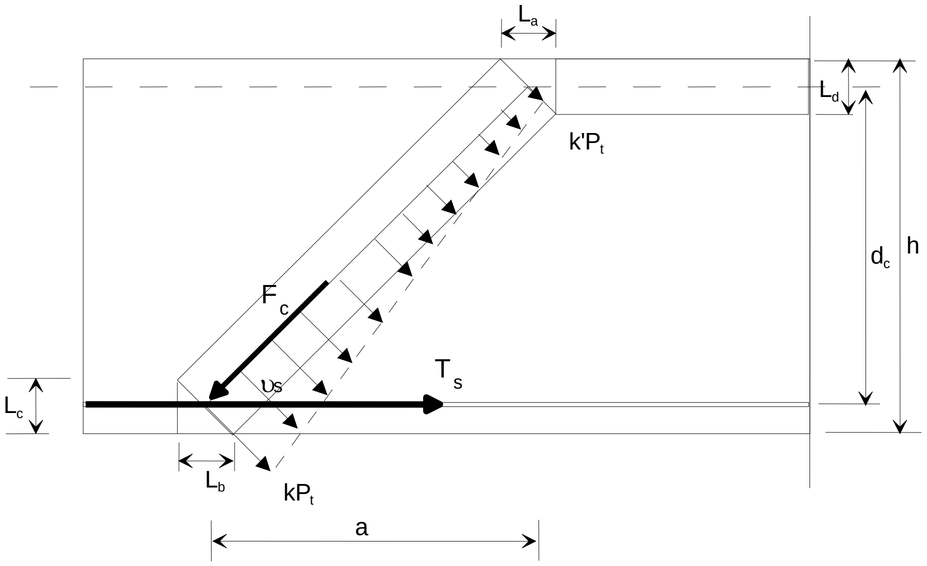

Figure (1) Assumed stress distribution due to bottom steel

$$\tan\theta_s = \frac{h - \frac{L_d}{2} - \frac{L_c}{2}}{a} \qquad ...(4)$$

where *h* is the beam depth, *d* is the effective depth, L_c and L_d are the respectively depths of bottom and top nodal zones,

$$L_c = 2(h - d) \qquad ...(5)$$

and *a* is the shear span measured from center lines between the load and support bearing plates. The term L_d is initial unknown. For convenience and simplicity, assuming $L_d = L_c$ gives an error less than 2% due to that L_d is typically ten times smaller than the beam height *h* [3,10].

The principal tensile stress *f1* at the bottom nodal zone arises from the component force of longitudinal reinforcement in the direction perpendicular to the diagonal strut, namely, $T_s.sin\Theta_s$ as follows

$$f_1 = \frac{k.T_s.\sin\theta_s}{A_c/\sin\theta_s} = k.P_t \qquad ...(6)$$

where P_t is the average equivalent tensile stress across the diagonal strut and A_c is the effective cross sectional area of the beam ($A_c = b_w.d_c$). *k* is a factor taking account of the non uniformity of the stress distribution. As shown in Fig. (2), considering one reinforcing bar that inclines at an angle θ_w from horizontal. From force equilibrium in the *f1* direction, the following equation can be established:

$$\left(\frac{k'+k}{2}\right).P_t.b_w.d_c/\sin\theta_s = T.\sin(\theta_s+\theta_w) \qquad ...(7)$$

Where from moment equilibrium about the top node, gives:

$$\left(\frac{k'}{2}+\frac{k-k'}{3}\right).(d_c/\sin\theta_s)^2.b_w.P_t = T.\sin(\theta_s+\theta_w).\frac{d_w}{\sin\theta_s} \qquad ...(8)$$

From equations (7) and (8), the factors k and k' can be obtained

$$k' = 4 - 6\frac{d_w}{d_c} \qquad \ldots (9a)$$

$$k = 6\frac{d_w}{d_c} - 2 \qquad ...(9b)$$

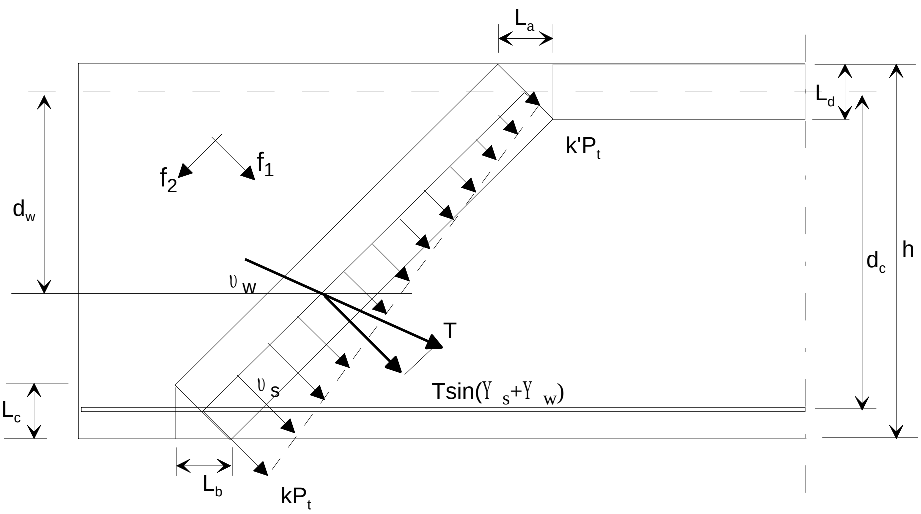

Figure (2) Assumed stress distribution due to one bar

For the case of bottom reinforcement, $d_w = d_c$, $\theta_w = 0$, the stress distribution factors $k' = -2$ and $k = 4$

For web reinforcement, assume that there are n_s web steel bars evenly distributed along the strut, the stress distribution factors can be written as below:

$$k' = \sum_{i=1}^{n_s} 4 - 6.\frac{d_{wi}}{d_c} = n_s \qquad ...(10a)$$

$$k = \sum_{i=1}^{n_s} 6.\frac{d_{wi}}{d_c} - 2 = n_s \qquad ...(10b)$$

In a similar fashion as Eq.(6), the tensile capacity, f_t, at the bottom nodal zone can be expressed as below:

$$f_t = f_{st} + f_{ct} \qquad \text{...(11)}$$

Where f_{st} represents the contribution from steel reinforcement, as below:

$$f_{st} = f_{ss} + f_{sw} \qquad \text{...(12)}$$

f_{ss} represents the contribution of bottom longitudinal steel, and can be calculated as

$$f_{ss} = \frac{4.A_s.f_y.\sin\theta_s}{A_c/\sin\theta_s} \qquad \text{...(13)}$$

f_{sw} represents the contribution of web reinforcement at the interface of nodal zone, and can be calculated as

$$f_{sw} = \frac{A_{sw}.f_{yw}.\sin(\theta_s+\theta_w)}{A_c/\sin\theta_s} \qquad \text{...(14)}$$

$$A_{sw} = ns.A_{sw1} \qquad \text{...(15)}$$

where A_{sw} represents the total area of web reinforcement crossing the concrete strut. For general case of vertical and horizontal web reinforcement, Eq. (14) can be written as below:

$$f_{sw} = \frac{A_{sv}.f_{yv}\sin 2\theta_s}{2.A_c} + \frac{A_{sh}.f_{yh}\sin^2\theta_s}{A_c} \qquad \text{...(16)}$$

A_{sv} and A_{sh} are the total areas of vertical and horizontal web reinforcement, respectively. According to ACI-318 [2], the concrete tensile strength can be calculated from the following relation:

$$f_{ct} = 0.56.\sqrt{f'_c} \qquad \text{...(17)}$$

In the cracked section, f_{ct} is respectively small ($f_{ct} \approx 0$) compared to tensile of reinforcement f_{st}. Eq. (11) can be rearranged as bellow:

$$f_t = \frac{4.A_s.f_y\sin^2\theta_s}{A_c} + \frac{A_{sv}.f_{yv}\sin 2\theta_s}{2.A_c} + \frac{A_{sh}.f_{yh}\sin^2\theta_s}{A_c} + f_{ct} \qquad \text{...(18)}$$

The principal compressive stress, f_2, in the direction of the strut at the bottom nodal zone can be computed by

$$f_2 = \frac{F_c - T\cos\theta_s}{A_{str}} \qquad \text{...(19)}$$

where A_{str} is the cross sectional area of strut at the bottom nodal zone and is defined as following:

$$A_{str} = b_w.(L_c.\cos\theta_s + L_b.\sin\theta_s) \qquad \text{... (20)}$$

Substituting Eqs. (2) and (3) into Eqs. (19) and (6) and combining with Eq. (1), gives:

$$V_n = \frac{1}{\frac{4.\sin\theta_s.\cos\theta_s}{A_c.f_t} + \frac{\lambda.\sin\theta_s}{A_{str}.f'_c}} \qquad \text{...(21)}$$

2-2-2 Top Nodal Zone

The top nodal is subjected to a biaxial compression-compression stress state, the failure mode is

$$\frac{f_2}{f'_c} = 1 \qquad \text{...(22)}$$

So, substitute Eq. (2) into Eq. (19) and combining with Eq. (22), gives:

$$V_n = f'_c.A_{str}.\sin\theta_s \qquad \text{...(23)}$$

2-2-3 General Nodal Zone

In general, the following proposed method can be formulated for any nodal zone in statically determined or statically undetermined truss. The internal forces of the truss can be found by assuming linear elastic material for each of concrete and steel bars [10,11]. Fc and T values denoted respect to external forces and the equilibrium of the nodal zones are found in the similar method of the nodal zone at bottom.

The shear strength of the deep beams takes as the smaller value from all of the nodal zones.

3. Verification Study

A number of tests are performed to evaluate the accuracy and robustness of the present method. The results obtained are also compared with those available in the literature to validate the present method. Although the tests are few in numbers, but they are able to display most of the parameters which affect on the accuracy.
The parameters include the effect of shear span to depth ratio (a/h), longitudinal bar anchorages, statically determined and undetermined trusses, and shear reinforcement on the predicated shear strength.

3-1 Aguilar's Deep Beam Test [12]

The deep beam used for this example was originally tested by Aguilar et al. [12]. The dimensions of the deep beam are given in **Figure (3)**. This example has shear span to depth ratio ($a/h = 1$) and can be represented by statically determinate truss. For this example, several strut-and-tie models were developed and evaluated using the strut-and-tie provisions for each of the specifications in order to predict the load capacity of the deep beam. In total, five models were analyzed by Martin and Sanders **[13]**, as shown in **Figure (4)**. The calculated capacities were then compared to the experimental capacity of 1286 kN. The results of the present method MSST ($\lambda = 0.8$ and F_{ct} as in **Eq.(17)**) and the results of different codes (as cited by **Ref. [13]**) are shown in **Table (1)**. The strut-and-tie model represented in **Figure (3)** is used for the following method and for the method of Zhang & Tan **[10]**. This method is more accurate and simply for representing the strut-and-tie model in comparison with the other methods.

Table (1) Results of different STM methods in analysis of Aguilar's deep beam

Methods	Model 1	Model 2	Model 3	Model 4	Model 5
AASHTO LRDF [14]	1.44	1.31	1.30	1.25	1.20
CSA A23.3 [7]	1.44	1.31	1.30	1.25	1.20
ACI 318 [2]	1.59	1.31	1.30	1.33	1.33
1999 FIB Rec [15]	1.59	1.31	1.30	1.33	1.33
DIN 1045-1[16]	1.80	1.41	1.30	1.34	1.33
Zhang & Tan [10]			1.40		
Present MSST			1.10		

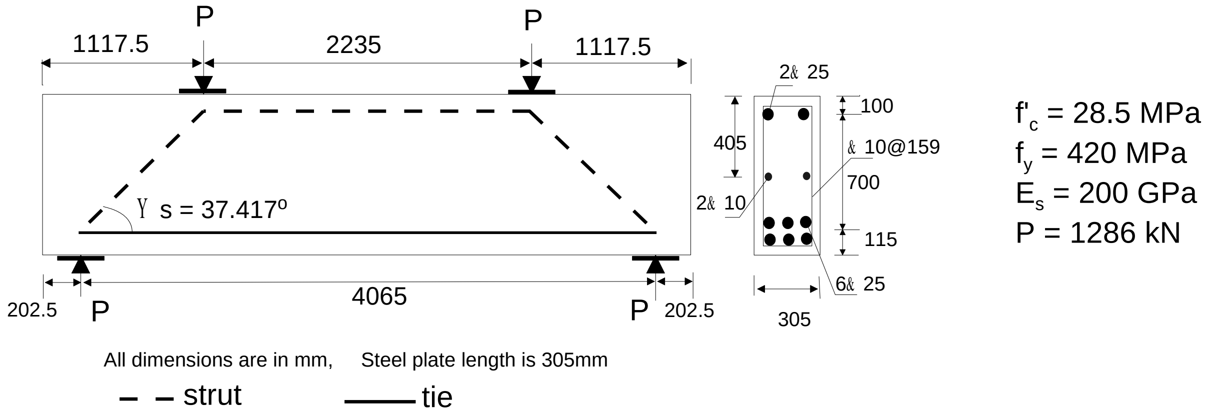

Figure (3) Dimensions of Aguilar's deep beam

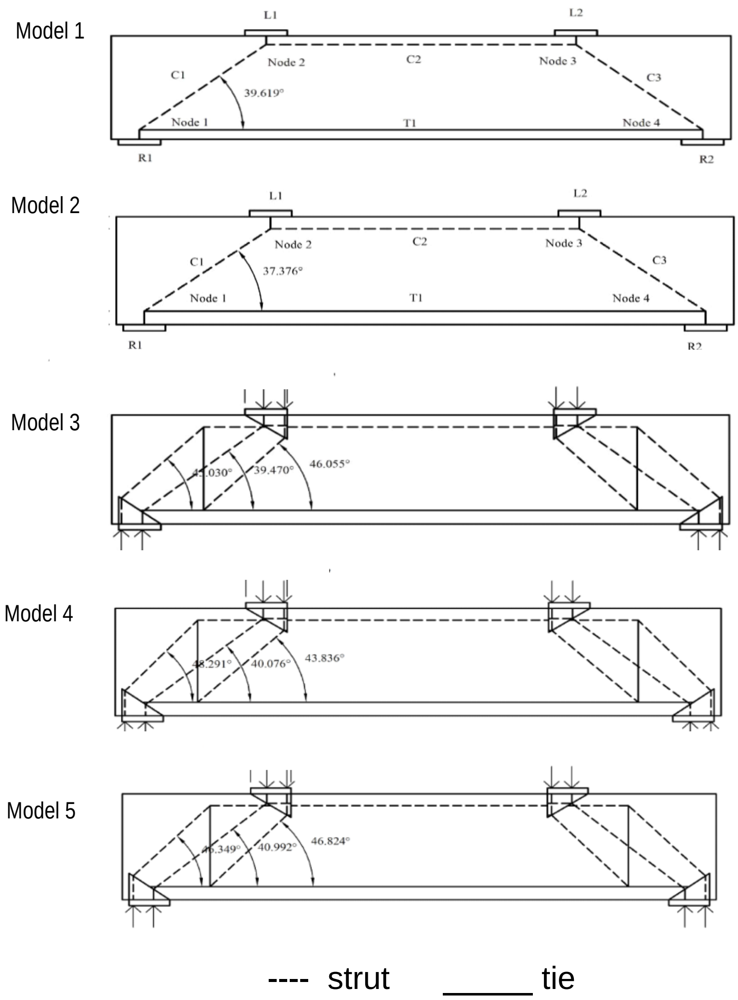

Figure (4) Geometry of different models used in analysis of Aguilar's deep beam [13]

3-2 Nathan and Brena Deep Beam Test [17]

To develop an acceptable strut-and-tie model for the analysis of deep beams, one must understand and identify the way shear force is transferred in deep beams. Beams with an *a/d* of 1.0 or below transfer shear force primarily through formation of a tied arch mechanism [17], where concrete diagonal struts form between the point of application of load and the support. A horizontal tie is needed to anchor these struts at their base and preserve horizontal force equilibrium at nodes located over the supports. While for beams with an *a/d* of 2.0 or greater transfer shears force primarily through formation of a truss mechanism. The top and bottom chords correspond to the compression stresses and longitudinal reinforcement of the beams, respectively. Web members in the truss model are made up of vertical ties and diagonal struts to complement the shear force transfer in the beam.

The dimensions, properties and the proposed strut-and-tie model of the following example are shown in **Figure (5)**. This example has shear span to depth ratio ($a/h = 2$) and can be represented by statically determinate truss.

The experimental specimens show the crack pattern, all the cracks initiated as vertical cracks regardless of region in the beam. This behavior is consistent with a truss mechanism for load transfer.Three different anchorage lengths were used, ranging as 43%, 50%, and 75%of the development length computed according to ACI code [2].

The results of experimental tests and the present method for failure load (P) are tabulated in **Table (2)**. The present method MSST ($\lambda = 0.8$ and F_{ct} as in **Eq.(17)**) gives accurate results and the formulation that used in this method is based on the full anchorage.

Table (2) Failure Loads of Nathan and Brena deep beams

Specimen (anchorage%)	Eperimental Load P_{Exp} (kN)	Present MSST P_{MSST} (kN)	P_{Exp}/P_{MSST}
43	313	308.2	1.02
50	297		0.96
75	266		0.86

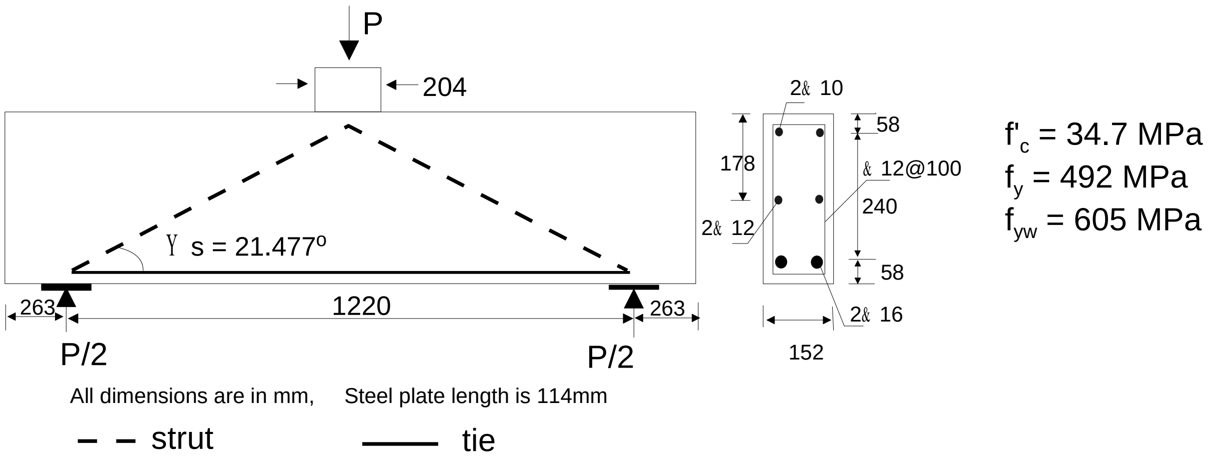

Figure (5) Details of Nathan and Brena deep beams

3-3 Yang et al. Continuous Deep Beams [11]

The results of this study show that the load transfer capacity of shear reinforcemt was much more prominent in continuous deep beams than in simply supported deep beam and the horizontal shear reinforcement was always more effective than vertical shear reinforcement [11]. The load transferred to the end and intermediate support against the total applied load was read by load cells. Good agreement between the results of linear elastic analysis with the experimental support reactions against the total applied load in all beams [11].

Figure (6) shows the details of continuous beam and the internal truss analysis forces. Four different web reinforcement was proposed for horizontal and verical shear reinforcement. **Table (3)** shows the details of shear reinforcement, and the experimental and theoretical results of failure load (P). As fixed by Yang et al. [11] the horizontal shear reinforcement is more important than the vertical reinforcement in behavior of continuous deep beams, the pressent MSST by using ($\lambda = 0.8$ and F_{ct} as in **Eq.(17)**) was accurately for model **C** and **D** while the results of ($\lambda = 1.0$ and $F_{ct} = 0$) are agrement with the model **A** and **B**. The following example explains the accuracy of the present MSST for continuous deep beams (statically indeterminate) has shear span to depth ratio ($a/h = 0.5$) and also the effect of shear reinforcemet on the predected shear strength

Table (3) Failure loads for Yang et al. deep beams

Model	Shear reinforcements (mm)		Experimental load	ACI 318-05	Pressent MSST
	Stirrups	Horizontal bars	P_{Exp} (kN)	P_{Exp}/P_{ACI}	P_{Exp}/P_{MSST}
A	-	-	1635	1.260	1.020
B	Φ6@60	-	1789	1.378	1.056
C	Φ6@120	Φ6@120	2117	1.305	1.020
D	-	Φ6@60	2317	1.785	1.013

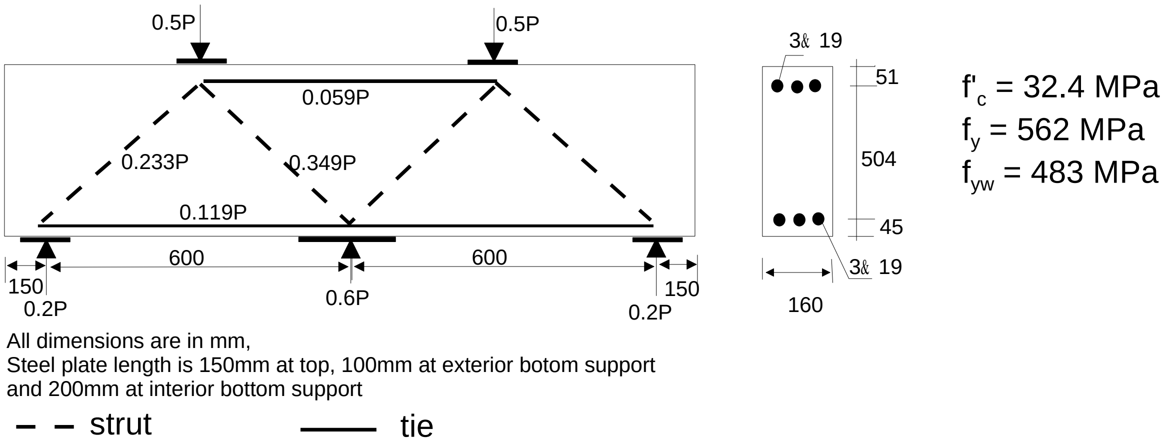

Figure (6) Details of Yang et al. deep beams

4. Conclusions

By considering a simplified mechanical behavior of deep beams and proposing model to determine the shear strength, after comparison with different tests available in the literature, the following conclusion can be drawn,

1. Each of force and moment equilibrium which was satisfied in this formulation and also the stress distribution factor *k*.
2. Strain compatibility and concrete softened was satisfied by using modified Mohr-Coulomb failure mode, that which was modified according to Kupfer and Gerstle study to represent concrete material.
3. The proposed model MSST for design and analysis of deep beams yields to a simple formula having a physical significance of the different cases of deep beams considered in this investigation.
4. The proposed MSST is applicable to analysis deep beams with various parameters, such as shear span to depth ratio (*a/h*), longitudinal bar anchorages, statically determined and undetermined trusses, and shear reinforcement.
5. In comparison with other models, the proposed model has good accuracy and validation for all tests examples.

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