Difference between the effect of SiO_2 fiber and GeO_2 fiber on the fiber parameters using Sellmeier formula

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Abstract

The material of fiber and its length affect on its characteristics such as chromatic dispersion $D(\lambda)$ and bit error rate (BER). So in this paper two types of glasses, the fused silica (SiO_2) and fused Germania (GeO_2) were considered to study these effects and find which fiber is better to use in optical communications for wavelength in the range (1-2) µm. Chromatic dispersion comes from combining two dispersions (waveguide dispersion and material dispersion). To solve the material dispersion, Sellmeier formula was used. The numerical results and the figures shown in this paper were obtained by using Matlab software 2008. As a result of this paper, the GeO_2 fiber is better than SiO_2 fiber in communications because it has higher BER and less $D(\lambda)$, and the fiber length inversely proportion to the BER.

الخلاصة

مادة الليف وطوله يؤثران على خصائصه مثل التشتت اللوني $D(\lambda)$ ونسبة خطأ الارقام الثنائية (BER). لذلك في هذا البحث تم اعتبار نوعين من الزجاج، السيليكا المصهورة (SiO_2) والجرمانيا المصهورة (GeO_2) لدراسة هذه التأثيرات وايجاد اي ليف افضل للاستخدام الاتصالات البصرية لطول موجي بمدى (١-٢) µm . التشتت اللوني يأتي من جمع تشتتين (تشتت موجه الموجة وتشتت المادة). لحل تشتت المادة، تم استخدام معادلة Sellmeier. النتائج العددية والرسوم الموضحة في هذا البحث تم الحصول عليها باستخدام برنامج Matlab 2008. كنتيجة لهذا البحث، ليف GeO_2 افضل من ليف SiO_2 في الاتصالات لانه يملك BER اعلى و $D(\lambda)$ اقل، وطول الليف يتناسب عكسياً مع BER.

1- Introduction

Since SiO_2 is the most fundamental substance in the earth crust and plays an important role in modern solid state technology, many theoretical calculations concerning SiO_2 polymorphs have been performed with focus on the crystal structure. SiO_2 has the most dispersive bandwidth due to the enhanced covalency and hybridization compared to GeO_2. [1]

Germania glass was prepared from high purity GeO_2 powder. Refractive index dispersion was used to calculate material dispersion and to provide a model for representing the dispersion of GeO_2-SiO_2 glasses. James [2] found the wavelength of zero material dispersion to be in agreement with theoretical calculation. Model propagation is modeled for a GeO_2 core-silica clad fiber. Results support compositional dependence of profile dispersion in GeO_2-SiO_2 fibers. [2]

Chromatic dispersion is the variation in the speed of propagation of a light wave signal with wavelength [3]. It leads to spreading of the light pulses and eventually to inter-symbol-interference (ISI) with increased bit error rate (BER).

The origin of chromatic dispersion in single mode fibers is a result from the interplay of material dispersion and wave-guide dispersion. [4]

For low bit rates the spectral width of the light pulses is small. Thus the spreading of the pulses is weak. Also the distance between two light pulses is large. This means that it needs a very long fiber link with high chromatic dispersion in order to make the pulses overlap. For higher bit rates this is different. On the one hand a signal has a much broader bandwidth, so the pulse spreading is larger for the same amount of dispersion. On the other hand the light pulses are closer together so that the overlapping of the pulses starts with less spreading. The consequence of both effects combined is that the dispersion limit is inverse proportional to the square of the bit rate. [5]

$$\text{Dispersion limit}\,\alpha\,\frac{1}{Bitrate^2} \qquad (1)$$

Its unit is ps/(nm-km). This parameter tells us how many picoseconds the pulse broadens per kilometer of fiber per nanometer of pulse spectral width. [6]

Taras [7] shows analytically if the response of a signal reshaping processor is slower than (or comparable to) the time scale of variations of the temporal profile of the input signal, then such a processor necessarily degrades the signal's bit error rate (BER). He made the BER comparison for the cases where the receiver is placed immediately before and immediately after the processor. [7]

Abdul Gafur, Doru Constantinescu, and Md Dulal Haque [8] found the effect of dispersion of fiber considering m-Sequence Optical Code Division Multiple Access (OCDMA) network based on star coupler and optical receiver. Matlab simulations performed to find the limitations imposed by dispersion on the number of user and length of transmission of fiber. As a result they investigated the performance curve for bit error rate (BER), signal to noise ratio (SNR) and eye diagram. In the curve of bit error rate, they found that the error is decreased when the number of simultaneously users increased. In the performance curve of SNR, they also observed that the system performance is increased with the raising SNR. [8]

2- Theoretical Basics

Maxwell's equations predict the propagation of electromagnetic energy away from time-varying sources (current and charge) in the form of waves. Consider a linear, homogeneous, isotropic media characterized by (μ, ε, σ) in a source-free region. We start with the source-free, instantaneous Maxwell's equations written in terms of E and H only. Note that conduction current in the source-free region is accounted for in the σE term.

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \qquad (2)$$

$$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t} \qquad (3)$$

$$\nabla \cdot E = 0 \qquad (4)$$

$$\nabla \cdot H = 0 \qquad (5)$$

Taking the curl of eq. (2)

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}(\nabla \times H) \qquad (6)$$

Substituting (3) into (6):

$$\nabla \times \nabla \times E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla \times \nabla \times E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad (7)$$

Taking the curl of eq. (3)

$$\nabla \times \nabla \times H = \sigma(\nabla \times E) + \varepsilon \frac{\partial}{\partial t}(\nabla \times E) \qquad (8)$$

and substituting (2) into (8)

$$\nabla \times \nabla \times H = \sigma\left(-\mu \frac{\partial H}{\partial t}\right) + \varepsilon \frac{\partial}{\partial t}\left(-\mu \frac{\partial H}{\partial t}\right)$$

$$\nabla \times \nabla \times H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} \qquad (9)$$

Using the vector identity

$$\nabla \times \nabla \times F = \nabla(\nabla \cdot F) - \nabla^2 F$$

in (7) and (9) gives

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad (10)$$

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H = -\mu\sigma \frac{\partial H}{\partial t} - \mu\sigma \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2} \qquad (11)$$

$$\nabla^2 H = \mu\sigma \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

For time-harmonic fields, the instantaneous (time-domain) vector is related to the phasor (frequency-domain) vector by, the instantaneous vector wave equations are transformed into the phasor vector wave equations:

$$\nabla^2 E_s = \mu\sigma(j\omega)E_s + \mu\varepsilon(j\omega)^2 E_s = j\omega\mu(\sigma + j\omega\varepsilon)E_s \qquad (12)$$

$$\nabla^2 H_s = \mu\sigma(j\omega)H_s + \mu\varepsilon(j\omega)^2 H_s = j\omega\mu(\sigma + j\omega\varepsilon)H_s$$

Let

$$j\omega\mu(\sigma + j\omega\varepsilon) = \beta^2$$

the phasor vector wave equations reduce to

$$\nabla^2 E_s - \beta^2 E_s = 0 \tag{13}$$

$$\nabla^2 H_s - \beta^2 H_s = 0$$

The complex constant β (is defined as the propagation constant (m^{-1})).

$$\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\gamma \tag{14}$$

The real part of the propagation constant (α) is defined as the attenuation constant (Np/m) while the imaginary part (γ) is defined as the phase constant (rad/m). The attenuation constant defines the rate at which the fields of the wave are attenuated as the wave propagates. An electromagnetic wave propagates in an ideal (lossless) media without attenuation (α= 0). The phase constant defines the rate at which the phase changes as the wave propagates.
Given the properties of the medium (μ, ε, σ), we may determine equations for the attenuation and phase constants. [9, 10]

$$\beta^2 = j\omega\mu(\sigma + j\omega\varepsilon) = (\alpha + j\gamma)^2 = \alpha^2 + j2\alpha\gamma - \gamma^2$$

$$\operatorname{Re}\beta^2 = \alpha^2 - \gamma^2 = -\omega^2\mu\varepsilon$$

$$\operatorname{Im}\beta^2 = 2\alpha\gamma = \omega\mu\sigma$$

Solving equation (14) for α & γ, to get:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]} \tag{15}$$

$$\gamma = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]} \tag{16}$$

The relation between waveguide dispersion $D_w(\lambda)$ and the effective refractive index n_{eff} of a guided mode in Photonic Crystal Fiber PCF is as Eq. (17) [11]:

$$D = -\frac{\lambda}{c} \times \frac{d^2(n_{eff})}{d\lambda^2} \qquad (17)$$

$$n_{eff}(\lambda) = \frac{\beta(\lambda, n_m)}{k_0} \qquad (18)$$

Where β is the propagation constant that calculated from equation (14), n_m is the refractive index of pure silica SiO_2 (or any material is chosen), is the wave number. In order to utilize the scaling transformation of PCF, the chromatic dispersion $D(\lambda)$ of a PCF can be approximated by a sum of waveguide dispersion and material dispersion:

$$D(\lambda) = D_w(\lambda) + \Gamma(\lambda) D_m(\lambda) \qquad (19)$$

Where $\Gamma(\lambda)$ is the confinemental factor in silica, which is close to unity for most particular PCFs. The material dispersion D_m is decided by the material that used in the PCF.

For lossless media (α= 0), the waveguide dispersion will equal to zero so the chromatic dispersion will equal to material dispersion; i.e.:

$$D(\lambda) = D_m(\lambda) \qquad (20)$$

For the case of SiO_2, the material dispersion can by directly derived from the Sellmeier equation:[12]

$$n(\lambda) = \sqrt{1 + \frac{0.6961663\lambda^2}{\lambda^2 - (0.0684043)^2} + \frac{0.4079426\lambda^2}{\lambda^2 - (0.1162414)^2} + \frac{0.8974794\lambda^2}{\lambda^2 - (9.896161)^2}} \qquad (21)$$

For the case of GeO_2, the material dispersion can by directly derived from the Sellmeier equation:[12]

$$n(\lambda) = \sqrt{1 + \frac{0.80686642\lambda^2}{\lambda^2 - (0.06897261)^2} + \frac{0.71815848\lambda^2}{\lambda^2 - (0.1539661)^2} + \frac{0.85416831\lambda^2}{\lambda^2 - (11.841931)^2}} \qquad (22)$$

$$BER = \frac{1}{4 * D(\lambda) * L * \Delta\lambda} \qquad (23)$$

Where BER is the bit error rate, L is the fiber length, $\Delta\lambda$ is the spectral width of the light source. An especially MATLAB package is designed to evaluate these parameters through the simulation of equations (17, 20, 21, 22, 23). [13, 14]

3. Results and discussion

From the above equations and by using Matlab software, the numerical results shown in tables (1 & 2) were obtained, , these results represent chromatic dispersion D(λ) and bit error rate BER for different crystal fiber lengths L. Figures (1 & 7) show the relation between the bit error rate and chromatic dispersion for three lengths of fibers (50km, 100km & 200km) for SiO_2 and GeO_2 respectively, these figures explain the behavior of BER which is calculated as the number of bits that were in error, as a proportion of the total number of bits transmitted, or received, or processed over a given period of time with different crystal fiber lengths so that the error at the 200km length will be less than that of 50km length, because the relation between them is inversely proportional. These lengths were taken randomly but they are in the range that would used in optical communications. From figures (1 & 7), the BER of GeO_2 is greater than that of SiO_2 fiber. Figures (2 & 8) show the relation between chromatic dispersion and wavelength which will be in the range (1-2) µm for SiO_2 and GeO_2 respectively, these figures explain the chromatic dispersion of GeO_2 fiber is less than of SiO_2 fiber, that means the GeO_2 fiber is better in transmission than SiO_2 fiber. For wavelengths less than 1, D(λ) will decrease until it reaches zero at approximately 0.195µm for SiO_2 and keep decreasing till reaches its minimum value $-4.166*10^7$ ps/km.nm at approximately 0.12µm then return to zero at 0.101µm as shown in figure (4). While for wavelengths greater that than 2, D(λ) will increase to reach its maximum value 1275 ps/km.nm at 8.3µm as shown in figure (5). Then it will decrease again to its minimum value $-2.4067*10^{14}$ at 9.9µm and return to zero at 9.965µm as shown in figure (6). For wavelengths less than 1, D(λ) will decrease until it reaches zero at approximately 0.15µm for GeO_2 and keep decreasing till reaches its minimum value $-3.0882*10^7$ ps/km.nm at approximately 0.12µm then return to zero at 0.1µm as shown in figure (10). While for wavelengths greater that than 2, D(λ) will increase to reach its maximum value 660 ps/km.nm at 8.32µm as shown in figure (11). Then it will decrease again to its minimum value $-8.9081*10^{13}$ at 9.9µm and return to zero at 9.94µm as shown in figure (12). Figures (4,5,6,10,11 & 12) explain that the best range of the wavelength is (1-2)µm to study the behavior of D(λ) of the fibers. Figures (3 & 9) show the behavior of the three parameters D(λ), BER and L as a 3D relation for SiO_2 and GeO_2 respectively, these figures explain that the short fiber has BER value greater than that of long fiber so the short fiber is better in communication. Table (3) illustrates the comparison between SiO_2 and GeO_2. The calculations and numerical results were depend on the refractive index (n) of the two materials, so it is the main reason in the difference between SiO_2 and GeO_2.

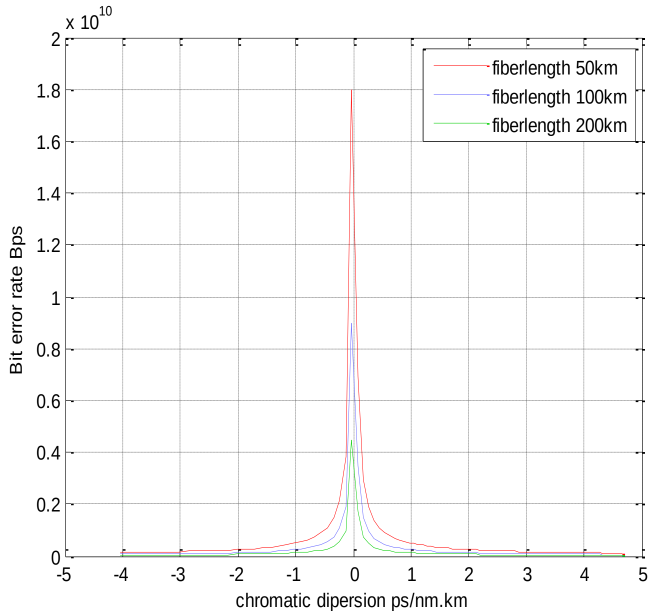

Figure (1) The relation between dispersion and Bit error rate of SiO_2 fiber

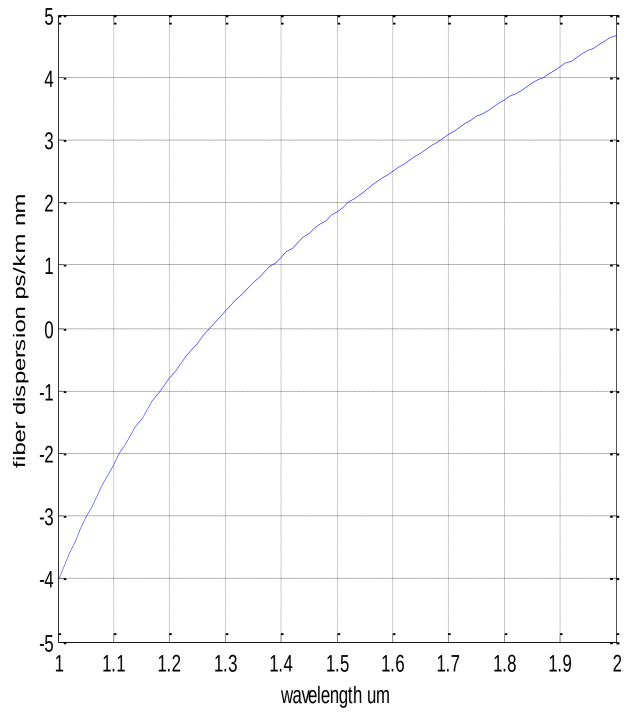

Figure (2) The relation between the wavelength and chromatic dispersion of SiO_2 fiber

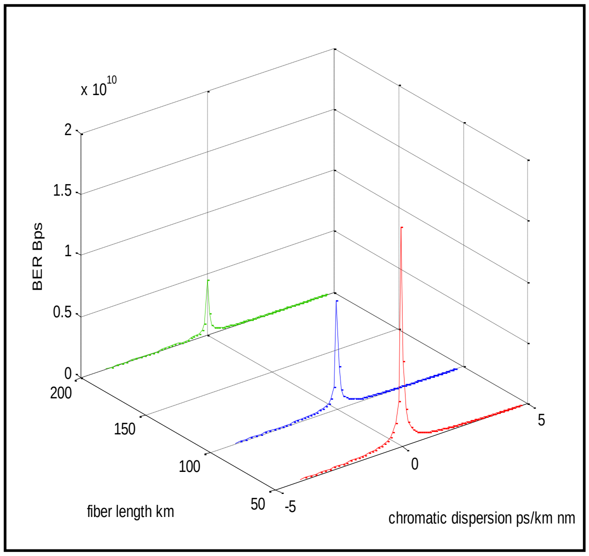

Figure (3) The relation in 3-D between chromatic dispersion and Bit error rate and fiber length of SiO_2 fiber

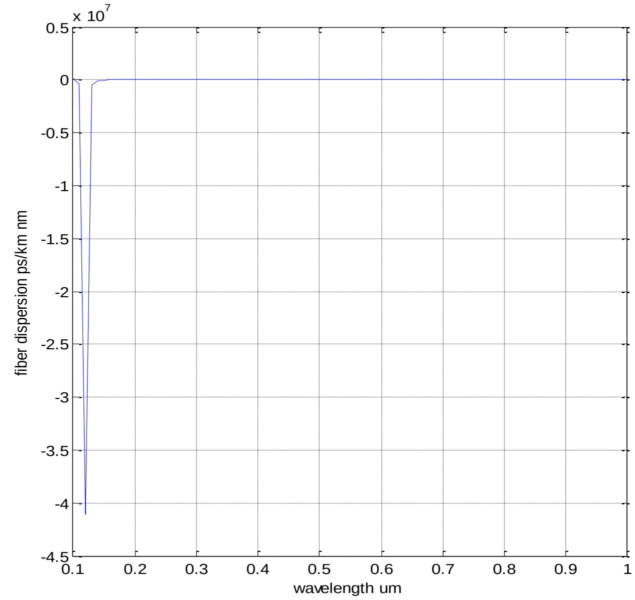

Figure (4) The minimum the D(λ)at range (0.1-1)μm of SiO_2 fiber

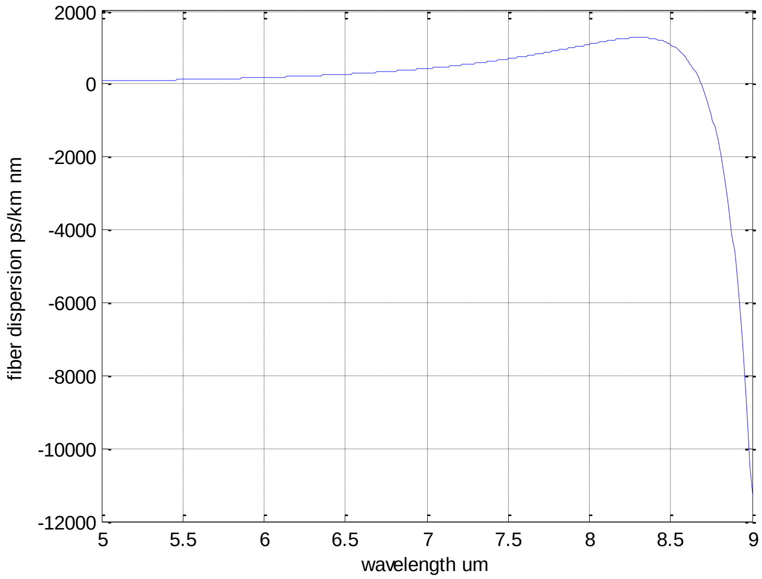

Figure (5) The maximum D(λ)at the range (5-9)µm of SiO_2 fiber

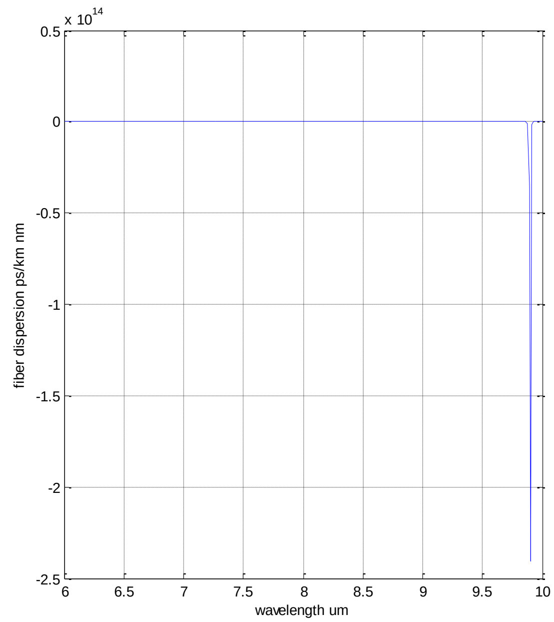

Figure (6) The minimum D(λ)at the range (6-10)µm of SiO_2 fiber

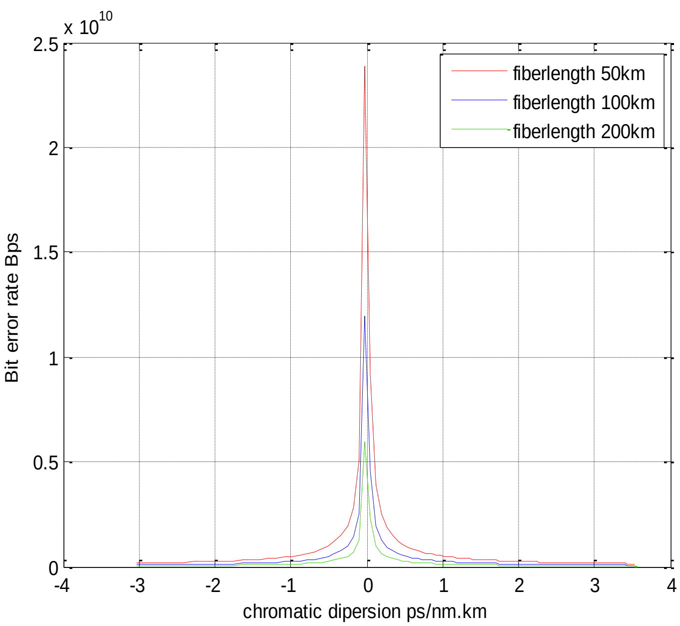

Figure (7) The relation between dispersion and Bit error rate of GeO_2 fiber

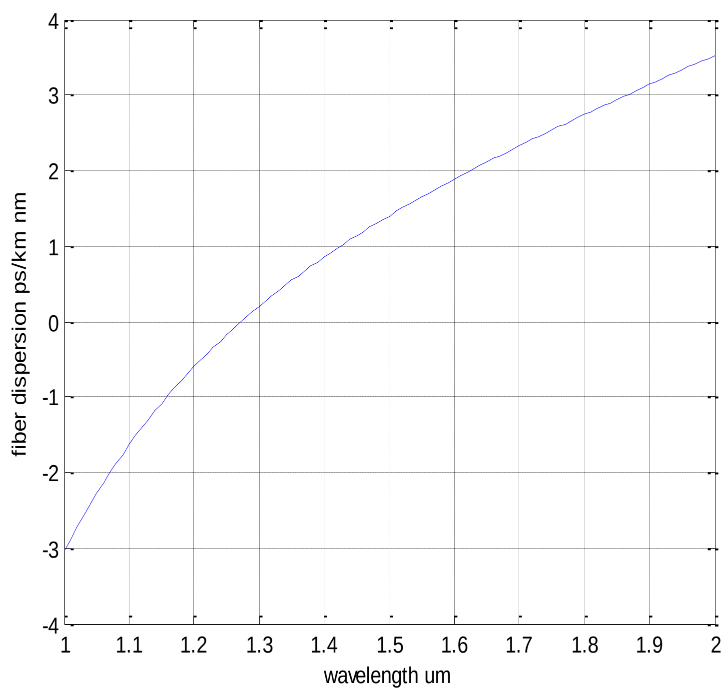

Figure (8) The relation between the wavelength and chromatic dispersion ofGeO_2 fiber

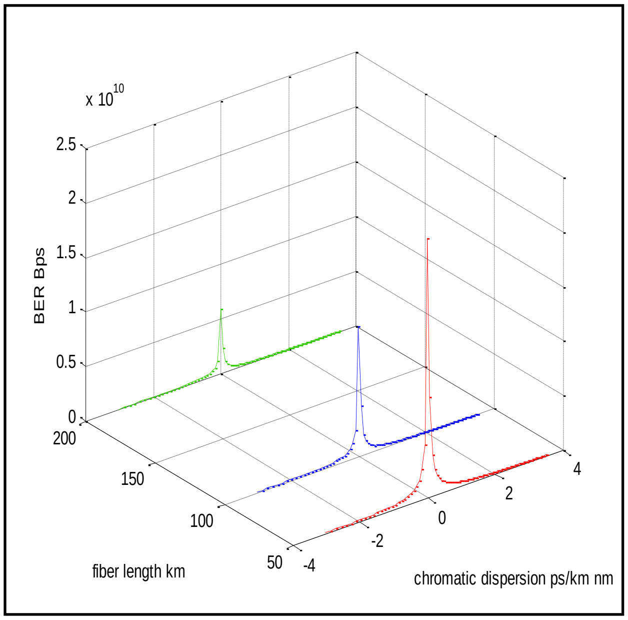

Figure (9) The relation in 3-D between chromatic dispersion and Bit error rate and fiber length of GeO_2 fiber

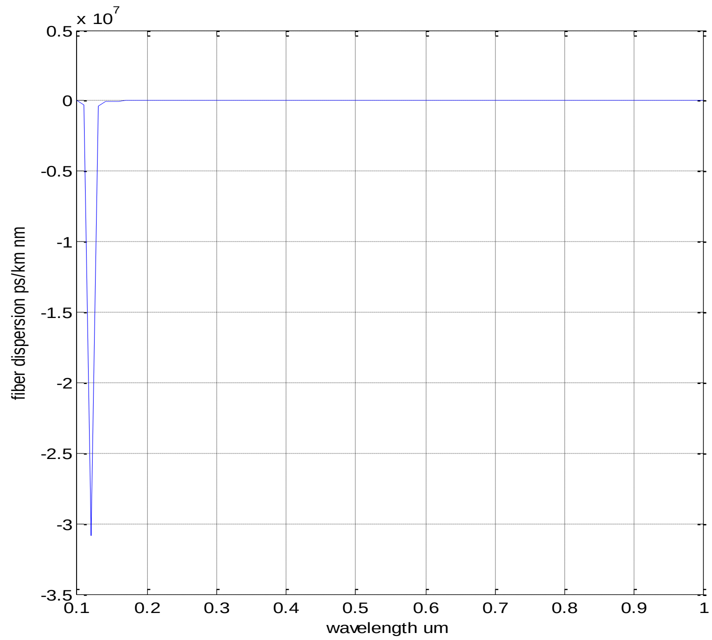

Figure (10) The minimum D(λ)at the range (0.1-1)μm of GeO_2 fiber

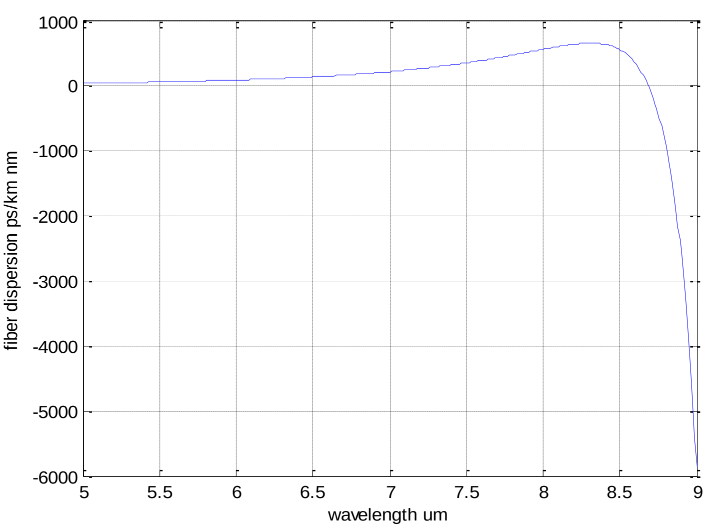

Figure (11) The maximum D(λ)at the range (5-9)μm of GeO_2 fiber

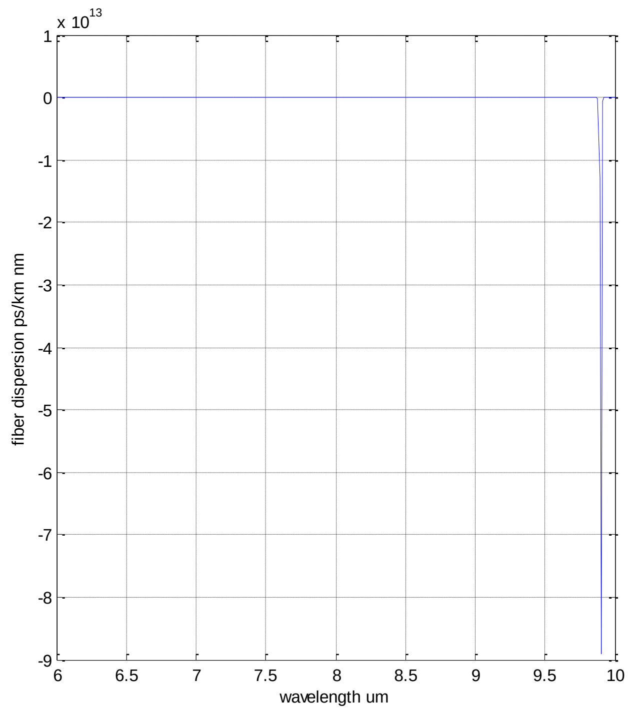

Figure (12) The minimum D(λ)at the range (6-10)μm of GeO_2 fiber

Table(1) Fused Silica (SiO_2) material fiber with n=1.48779

Wavelength λ(μm)	Chromatic dispersion(ps/km.nm)	BER(Gb/s km) at fiberlength=50km	BER(Gb/s km) at fiberlength=100km	BER(Gb/s km) at fiberlength=200km
1	-4.0348	0.0124	0.0620	0.0310
1.1	-2.1690	0.0231	0.1153	0.0576
1.2	-0.7967	0.0628	0.3138	0.1569
1.3	0.2660	0.1879	0.9397	0.4699
1.4	1.1285	0.0443	0.2215	0.1108
1.5	1.8589	0.0269	0.1345	0.0672
1.6	2.5016	0.0200	0.0999	0.0500
1.7	3.0865	0.0162	0.0810	0.0405
1.8	3.6344	0.0138	0.0688	0.0344
1.9	4.1602	0.0120	0.0601	0.0300
2	4.6753	0.0107	0.0535	0.0267

Table(2) Fused Germania (GeO_2) material fiber with n= 1.60846

Wavelength λ(μm)	Chromatic dispersion(ps/km.nm)	BER(Gb/s km) at fiberlength=50km	BER(Gb/s km) at fiberlength=100km	BER(Gb/s km) at fiberlength=200km
1	-3.0405	0.0164	0.0082	0.0411
1.1	-1.6345	0.0306	0.0153	0.0765
1.2	-0.6003	0.0833	0.0416	0.2082
1.3	0.2005	0.2494	0.1247	0.6235
1.4	0.8504	0.0588	0.0294	0.1470
1.5	1.4008	0.0357	0.0178	0.0892
1.6	1.8852	0.0265	0.0133	0.0663
1.7	2.3259	0.0215	0.0107	0.0537
1.8	2.7388	0.0183	0.0091	0.0456
1.9	3.1351	0.0159	0.0080	0.0399
2	3.5232	0.0142	0.0071	0.0355

Table(3) Comparison between (SiO_2) and (GeO_2) fibers

Parameters	SiO_2	GeO_2
BER(Gb/s km) at fiberlength=50km	1.8	2.389
BER(Gb/s km) at fiberlength=100km	0.9	1.1944
BER(Gb/s km) at fiberlength=200km	0.45	0.6
D(λ) (ps/km.nm)	4.675	3.523
Transmission at zero dispersion	at 1300 nm	at 1300 nm
Minimum D(λ) (ps/km.nm)	$-4.166*10^7$ at 0.12μm	$-8.9081*10^{13}$ at 9.9μm
Maximum D(λ) (ps/km.nm)	1275 at 8.3μ	660 at 8.32μm
Refractive index	1.48779	1.60846
The usage in optical communication	Familiar and well known	Better than SiO_2 in communication

4- Conclusion

1. The bit error rate (BER) for SiO_2 fiber is 1.8 Gb/s, while the BER for GeO_2 fiber is 2.389 Gb/s for fiber length 50 km.
2. The chromatic dispersion D(λ) for SiO_2 fiber is 4.675 ps/km.nm, while the D(λ) for GeO_2 fiber is 3.523 ps/km.nm.
3. The GeO_2 fiber is better than SiO_2 fiber in optical communications.
4. The transmission in both SiO_2 fiber and GeO_2 fiber was carried out with zero dispersion at 1300nm.
5. The minimum D(λ) for SiO_2 is $-4.166*10^{7}$ ps/km.nm at 0.12µm, while for GeO_2 is $-8.9081*10^{13}$ ps/km.nm at 9.9µm.
6. The maximum D(λ) for SiO_2 is 1275 ps/km.nm at 8.3µ, while for GeO_2 is 660 ps/km.nm at 8.32µm.
7. The best wavelength range to study the behavior of fiber parameters is (1-2)µm.
8. The bit error rate (BER) is inversely proportional to the fiber length L.
9. The short fiber is better in optical communication than the long fiber.
10. The chromatic dispersion D(λ) is inversely proportional to the square value of the bit rate.

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