Journal of Engineering and Development, Vol. 16, No.1, March 2012 ISSN 1813- 7822

Finite Element Analysis of Earth-retaining Structures Problems in Nonlinear Partially Saturated Soil

Dr. Hassanein F. Hassan
(lecturer), B.Sc., M.Sc., Ph.D Civil Engineering
Al-Mustansiriya University, College of Engineering, Department of Civil Engineering

Saif M. Jawad
(Assist. Lecturer), B.Sc., M.Sc. Civil Engineering
Baghdad University, College of Engineering, Engineering Consultancy Bureau

Abstract

This paper presents the finite element method to analyze the problems of earth-retaining structures in nonlinear partially saturated soil. The finite element formulation of the incremental Biot's theory (1941) of consolidation is presented. This formulation is extended to represent the nonlinear time dependent behavior of earth-retaining structures in partially saturated soil. Also the concept of the thin layer element is incorporated to simulate the behavior at the interfaces between the soil and retaining structures. The finite element method is applied to analyze the construction of real braced excavation in soft clay. The results obtained are compared to the observed field results. Good agreement is obtained and the numerical method proved how can be used it for analyzing related geotechnical problems involving consolidation and support systems in nonlinear partially saturated soil.

الملخص

هذا البحث يقدم طريقة العناصر المحددة في تحليل المنشأت الارضية الساندة في الترب اللاخطية والمشبعة الجزئيا. تم تطبيق هذه الطريقة على نظرية بايوت (١٩٤١) للانضمام وقد تم توسيع هذا التطبيق ليمثل التصرف الزمني اللاخطي للمنشأت الارضية الساندة في الترب المشبعة جزئيا. ان فكرة العنصر الطبقي الرفيع قد استخدمت لتمثل التصرف المتداخل بين التربة والمنشأت الساندة. تم استخدام طريقة العناصر المحددة في تحليل حفر مسند واقعي في التربة الطينية وقد تم مقارنة النتائج التي تم الحصول عليها مع النتائج الواقعية وهذه المقارنة اثبتت ان هنالك تطابقاً جيداً بين النتائج كما تبين ان الطريقة العددية يمكن استخدامها في تحليل مسائل الجيوتكنيك المشابه والمتضمنة مسائل الانضمام والانظمة الساندة في الترب اللاخطية والمشبعة جزئيا.

1. Introduction

The simulation of supported excavation by the finite element method is considered from the complex problems. Numerous procedures have been proposed for implementing the idea to simulate excavation using finite element method. This idea can be achieved by dividing the total excavation into a number of sequences. In the program it is assumed that after each sequence, the exposed surface can be treated as "stress free". In other words, normal stresses to the exposed surface are zero. Most researchers simulate the excavation in the finite element method using the linear or nonlinear elastic model because this model is simple in the application. In this paper the modified Cam clay model (Atkinson and Bransby (1978)) has been used to present soil model also the interface element between the structure and the soil which was proposed by Desai et al. (1984) was used to give more realistic results for a represented case. The finite element formulations of Biot's theory of consolidation Biot (1941) was used and extended by Small (2000) to include the problems of partially saturated soil in the case of high degrees of saturation at which the pore air pressure and pore water pressure are approximately equal. This formula of partially saturated soil was used because it is simple while the other formulations are very complex and need additional parameters from the laboratory test to be applied.

2. Modified Cam Clay Model

The stress space of this model can be represented in a three dimensional stress-specific volume space called the state boundary surface (S.B.S), as shown in Figure (1) (Konstantinos (2003)). Inside and on this surface a point representing the state of stress must lie.

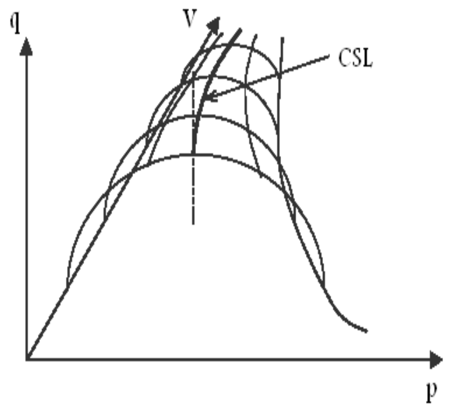


Figure (1): State boundary surface for modified Cam clay model. (after Konstantinos (2003))

In critical state theory the virgin compression, swelling and recompression lines are assumed to be straight in (ln(p'),v) plots with slopes (-λ and -κ) respectively, as shown in Figure (2) (Britto and Gunn (1987)). The equation of the isotropic virgin compression line is given as:

$$v = N - \lambda \ln(p') \quad \text{........ (1)}$$

where v is a specific volume, N is a constant for a particular soil and equal to v when ln(p')=0, i.e. p'=1, the value of N is given by the equation shown below:

$$N = \Gamma + (\lambda - \kappa)\ln(2) \quad \text{........ (2)}$$

where Γ represents the specific volume of soil on the critical state line p'=1kN/m^2.

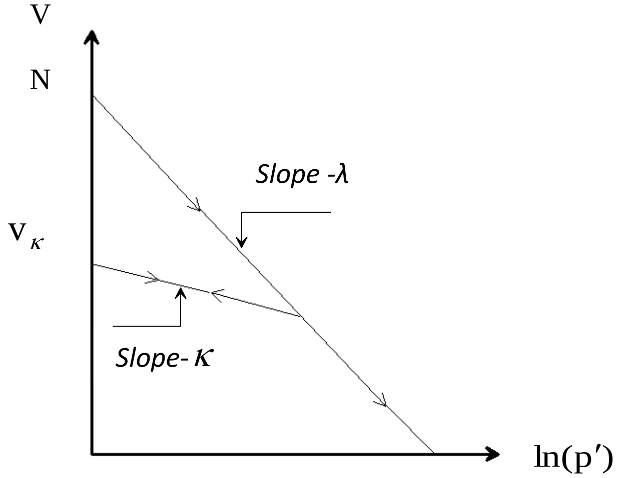


Figure (2): Identical (lnp',v) plot in critical state theory (after Britto and Gunn (1987))

If the sample of soil is subjected to isotropic compression stress, the point in v-ln(p) plane moves on the virgin compression line (λ-line), and if the load is removed, the point will be moved on the swelling line (κ-line). The equation of the κ-line is given as:

$$v = v_\kappa - \kappa \ln(p') \quad \text{........ (3)}$$

The initial state of the soil in a (p', q) plot is (p'_c, o) and when the sample of soil is subjected to a load in the standard triaxial test, soil is sheared and a point moves in the (p', v, q) space. The route from initial point to the point at the end applied is called effective stress path.
The mean stress at the critical state is calculated from (Britto and Gunn (1987)):

$$p_c = p + q^2 / p \cdot M^2 \quad \text{........ (4)}$$

$$p_{cs} = \frac{p_c}{2} \quad \text{........ (5)}$$

where M is a model parameter and p'_C is the isotropic yield stress. The volume at the initial stress and at the critical state is calculated from:

$$v_o = N - \lambda \ln\left(p'_c\right) + k \ln\left(p'_c / p_o\right) \quad \text{........ (6)}$$

$$v_{cs} = \Gamma - \lambda \ln\left(p'_{cs}\right) \quad \text{........ (7)}$$

3. Excavation in Modified Cam Clay

In contrast to linear analyses, the evolution of stresses in nonlinear analysis requires the numerical integration of the equilibrium equations over a finite time increment (Borja et al. (1989)).

$$w_{INT}(t) = w_{EXT}(t) \quad \text{...... (8)}$$

where

$$w_{INT}(t) = \int_{\Omega(t)} \nabla w \cdot \sigma \cdot d\Omega \quad \text{.... (9)}$$

represents the internal virtual work and

$$w_{EXT}(t) = \int_{\Omega(t)} w \cdot b d\Omega + \int_{\Gamma_T(t)} w \cdot T d\Gamma \quad \text{.... (10)}$$

represents the external virtual work.

Where: Ω is the problem domain, T surface traction and Γ_T is the boundary problem.

The constitutive equation for a nonlinear analysis is of the form:

$$\sigma^k_{n+1} = \sigma_n + \delta^k\left(\varepsilon_{n+1}, \varepsilon_n\right) \quad \text{.... (11)}$$

where σ_n and ε_n are the converged stress and strain vectors of the previous time step, respectively, and δ^k is the incremental stress function consistent with a given stress integration algorithm. The expression of nonlinear moduli obtained by taking the variation:

$$D^k_{n+1} \equiv \frac{\partial \sigma^k_{n+1}}{\partial \varepsilon^k_{n+1}} = \frac{\partial \delta^k\left(\varepsilon^k_{n+1}, \varepsilon_n\right)}{\partial \varepsilon^k_{n+1}} \quad \text{....... (12)}$$

4. Governing Equations for Fully and Partially Saturated Soils.

The constitutive relationship for saturated and unsaturated soils are assumed to be governed by two stress variables; effective stress in the soil skeleton and pore water pressure because the pore water pressures and pore air pressures are generally fairly close to each other for high degrees of saturation. This degree of saturation depends on the type of soil.
The complete behavior of the two phase skeleton - fluid state in fully saturated soils is governed by this equation (Lewis et al. (1976)):

$$\begin{bmatrix} K & L \\ L^T & -0.5\Delta t.H \end{bmatrix} \begin{Bmatrix} du^n \\ dp^n \end{Bmatrix} = \begin{Bmatrix} dF(\Delta t) \\ \Delta t.H.p^n(t) \end{Bmatrix} \text{........ (13)}$$

In partially saturated soils, a theoretical relationship between pore water pressure and degree of saturation was proposed by Lowe and Johnson (1960)

$$S_r = \frac{0.0099p + 0.98S_{r_o}}{0.98 + 0.0097p} \quad \text{........ (14)}$$

Where S_r is the degree of saturation at a pore water pressure p in kPa. S_{r_o} denotes the degree of saturation at zero pore water pressure.
Therefore the finite element equation becomes:

$$\begin{bmatrix} K & L \\ L^T & 0.5\Delta.5\Delta_s - Q \end{bmatrix} \begin{Bmatrix} du^n \\ dp^n \end{Bmatrix} = \begin{Bmatrix} dF(\Delta t) \\ \Delta t.H_s.p^n(t) \end{Bmatrix} \text{........ (15)}$$

It can be shown that the set of Equations (15) which has been developed for partially saturated soils has an additional matrix when compared to the set of equations for a saturated soil (Equation (13)). Matrix Q represents the change of storage of moisture with respect to change in water head (Small (2000)). For identification purposes, Q is called the storage matrix here,

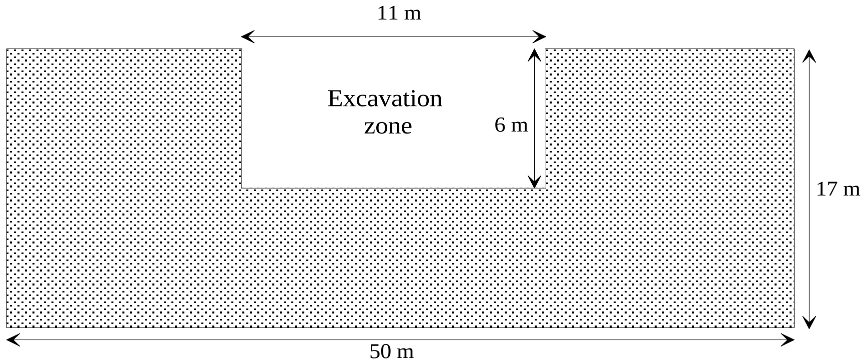
$$Q = \int_\Omega \frac{n\gamma_w}{S_r} \frac{\partial S_r}{\partial p} N_p \, N_p^T \, d\Omega \quad \text{........ (16)}$$

and H_s is the flow matrix which is also a function of degree of saturation.

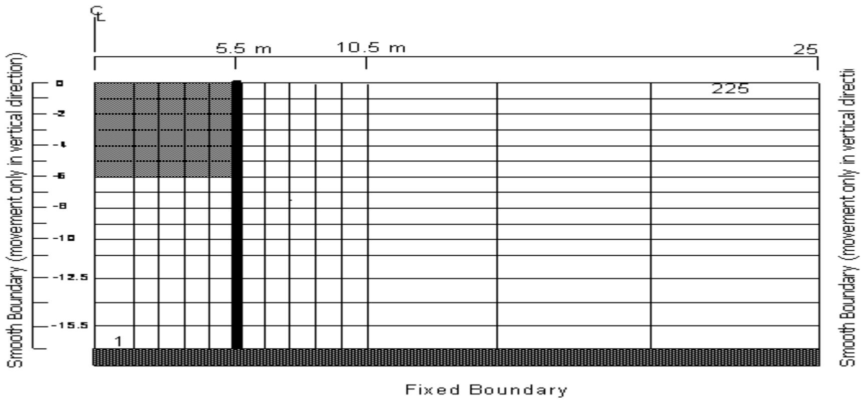
$$H_s = \int_\Omega \frac{1}{S_r} (\nabla.N_p^T)^T \frac{K_p}{\gamma_w} \nabla N_p^T \, d\Omega \quad \text{... (17)}$$

5. Two Dimensional Supported Excavations in Elasto-plastic Soil.

The excavation of a two dimensional opening in a soil where pore pressure dissipation occurs is a complex problem with moving drainage boundaries and changing geometry. This situation is well suited for analysis by the developed finite element program "EXCCONPSS"
In this research a finite element program called "EXCCONPSS" is developed by the authors to implement the modified Cam clay as a model of the soil and also the behavior of partially saturated soil.
The program is employed to analyze a typical supported excavation problem. The selected excavation cross section with excavation sequence and boundary conditions is as shown in Figure. (3a-b). The finite element mesh is composed of 225 eight-noded isoparametric soil elements, 15 three-noded isoparametric beam elements and one three-noded isoparametric bar (truss) element (in the case of supported wall with strut), the interface elements between the soil and the wall are used which depend on Desai et al. (1984) theory. The dimensions of the problem are 17 m depth and 50 m width and the dimensions of the excavation zone are 6m depth and 11m width. The depth of excavation is divided into six layers and each layer is of 1m depth and 11m width.



(a) Dimensions of the problem.



(b) Finite element mesh.

Figure (3): Problem details and finite element mesh.

The material properties used in the analysis are given in Table (1):

Table (1): The proposed material properties of soil and structure.

MATERIAL	MATERIAL PROPERTIES											
	E kN/m^2	ν	γ kN/m^3	$K_{\circ}$	G kN/m^2	n	M	κ	λ	Γ	Iz m^4/m	Ax m^2/m
Soil	10000	0.3	20.0	0.564	*	0.33	1.02	0.05	0.26	3.767	_	_
Interface	10000	0.3	20.0	0.564	Small value (slip mode) =10	0.33	_	_	_	_	_	_
Sheet pile	$2x10^8$	_	_	_	*	-	_	_	_	_	$1x10^{-4}$	0.1
Strut	$2x10^8$	_	_	_	*	-	_	_	_	_	_	1

*The value is calculated by the program.

Where $K_{\circ}$ is the coefficient of lateral earth pressure, and n is the porosity of the soil.

Two types of analysis are performed:

1- Supported excavation without strut.

In this case of supported excavation, three states of analysis are carried out based on the degrees of saturation of the soil. All the results are taken at the end of excavation. The coefficient of permeability used in all cases is ($1x10^{-4}$ m/day) for soft clay, and a rate of excavation of 0.15 m/day is used (this rate represents the realistic procedure of the work) to remove six layers of the soil.

In the first state, the soil is considered fully saturated soil (Sr = 100 %) and the analysis is based on the Equation (13). In the second and third states, the soil is considered partially saturated soil which has degree of saturation equal to (90 and 80 %) respectively. The analysis for both states is based on Equation (15). The distribution of the degree of saturation in the soil is based on Equation (14).

From Figure (4) it can be seen, that the state of saturated soil gives maximum wall deflection because friction in the soil is small. On the other hand, the state of partially saturated soil at degree of saturation (80 %) gives values of wall deflection less than in saturated soil especially at the surface because the degree of saturation is low and friction between the soil particles increases because the voids of the soil contain air so that the wall deflection is the smallest. Also, it can be seen, that the wall deflection are approximately the same for all states (Sr=80, 90 and 100%) in the lower part of the wall. From Figure (5) which represents the bending moment of the wall, it can be seen that the bending moment of the wall in the state of fully saturated soil is the greater than partially saturated soil because the lateral movement of the wall is greater.

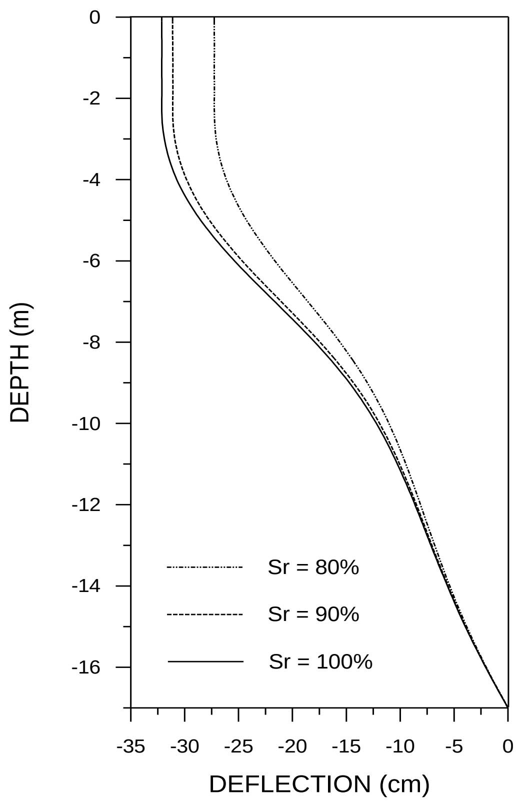


Figure (4): Lateral movement predicted of the wall for different degrees of saturation.

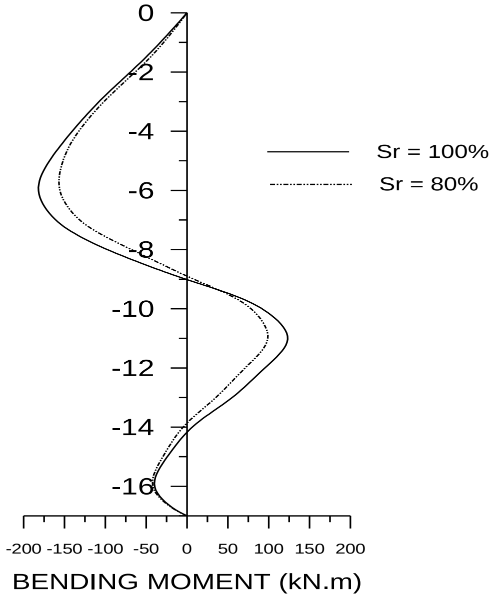


Figure (5): Predicted wall bending moment for different degrees of saturation.

Figure (6) represents the vertical surface movement behind the wall. Also, from this figure it appears that the settlement of fully saturated soil is greater than partially saturated soil because in the state of partially saturated soil, friction between the wall and the soil and between the soil particles is the greater and the excess pore water pressure is the smallest.

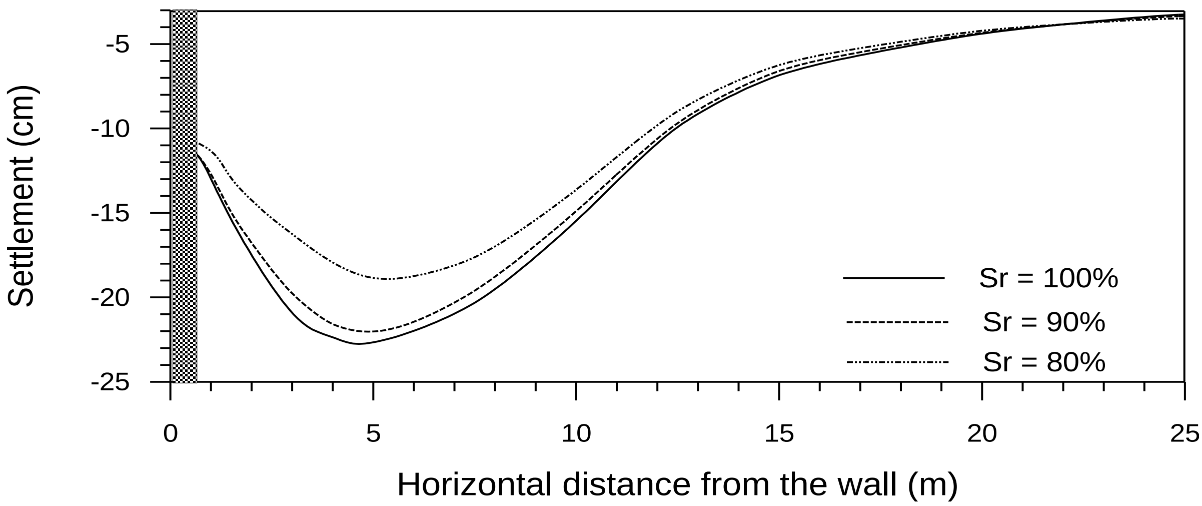
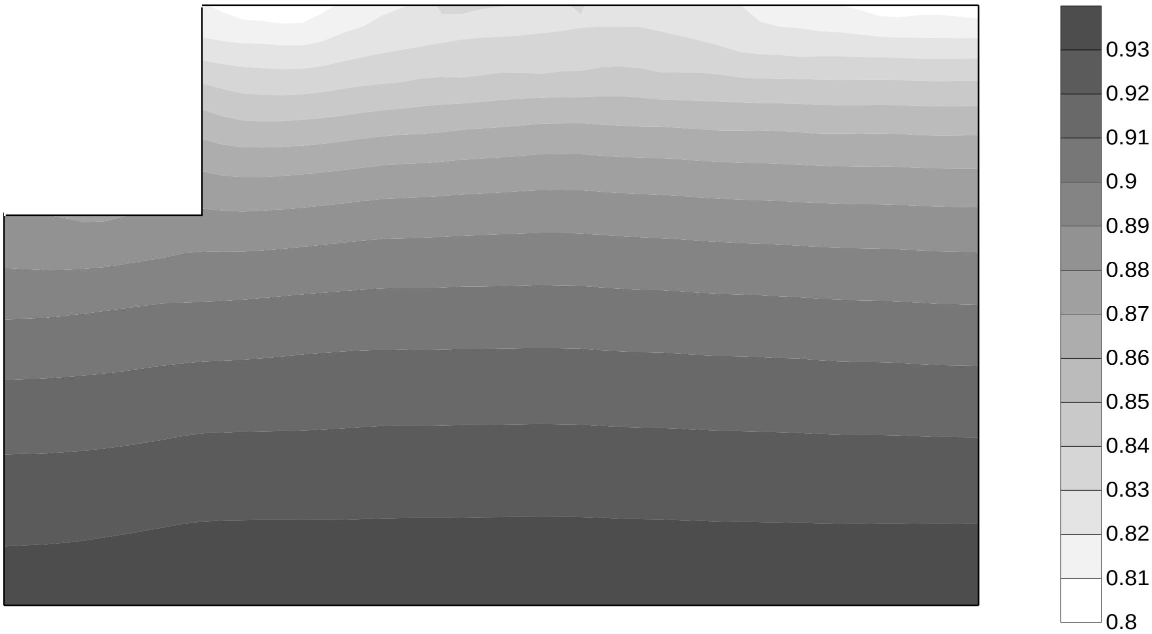
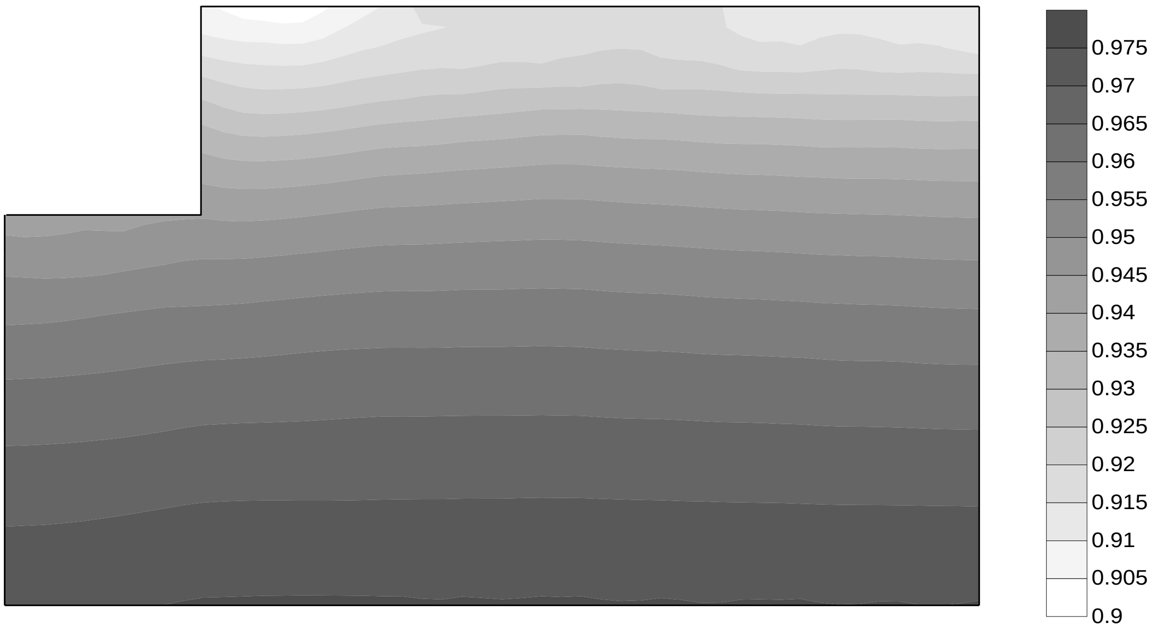


Figure (6): Surface movement predicted of the soil for different degrees of saturation.

Figure (7) represents the distribution of the degree of saturation in the soil. It can be seen that the degree of saturation increases with pore water pressure.



a): Sr = 80 %



b): Sr = 90%

Figure (7): Distribution of degree of saturation in the soil at the end of excavation.

Figure (8) represents the shear stresses generated at the end of excavation. These stresses are concentrated beside the wall in these two states but their values are different. The reason is that the excess pore water pressure in partially saturated soil is less than fully saturated soil which makes the shear stresses are greater.

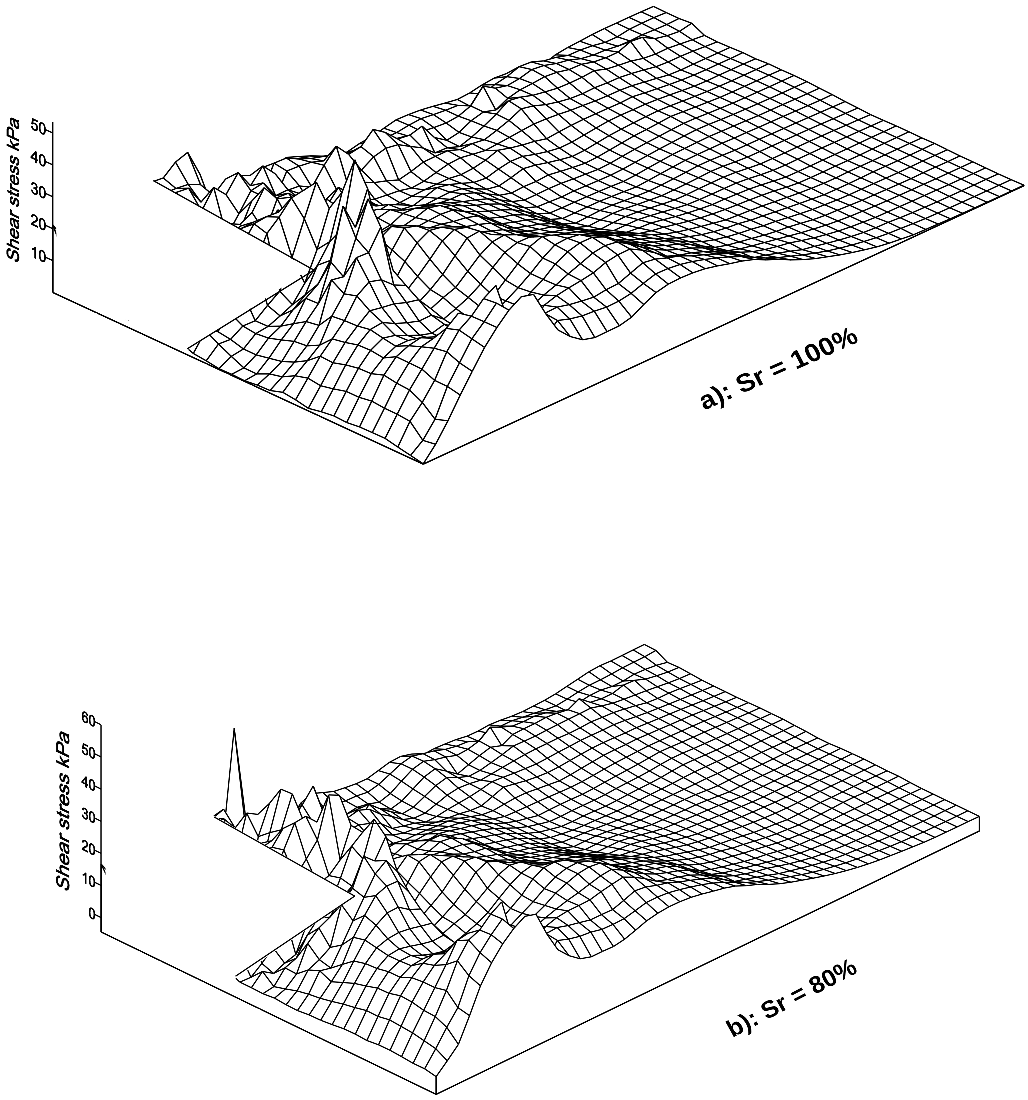
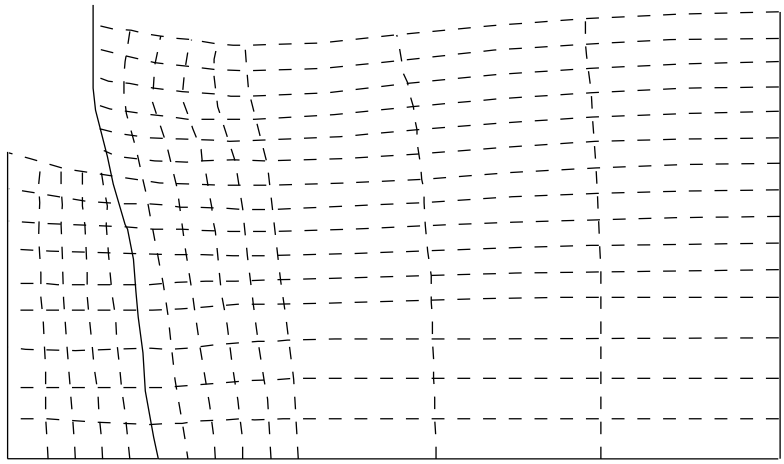
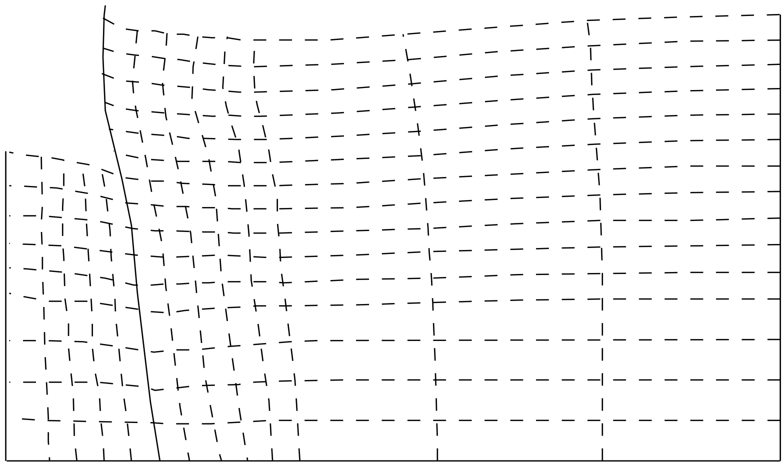


Figure (8): Predicted shear stresses in the soil at the end of excavation.

Figure (9) represents the deformed shape of the soil at the end of excavation when the degrees of saturation are 100% and 80%. All displacements are exaggerated by a factor of 10. From this figure it can be seen that the deformation in the case of fully saturated soil larger than partially saturated soil.



Sr = 100 %



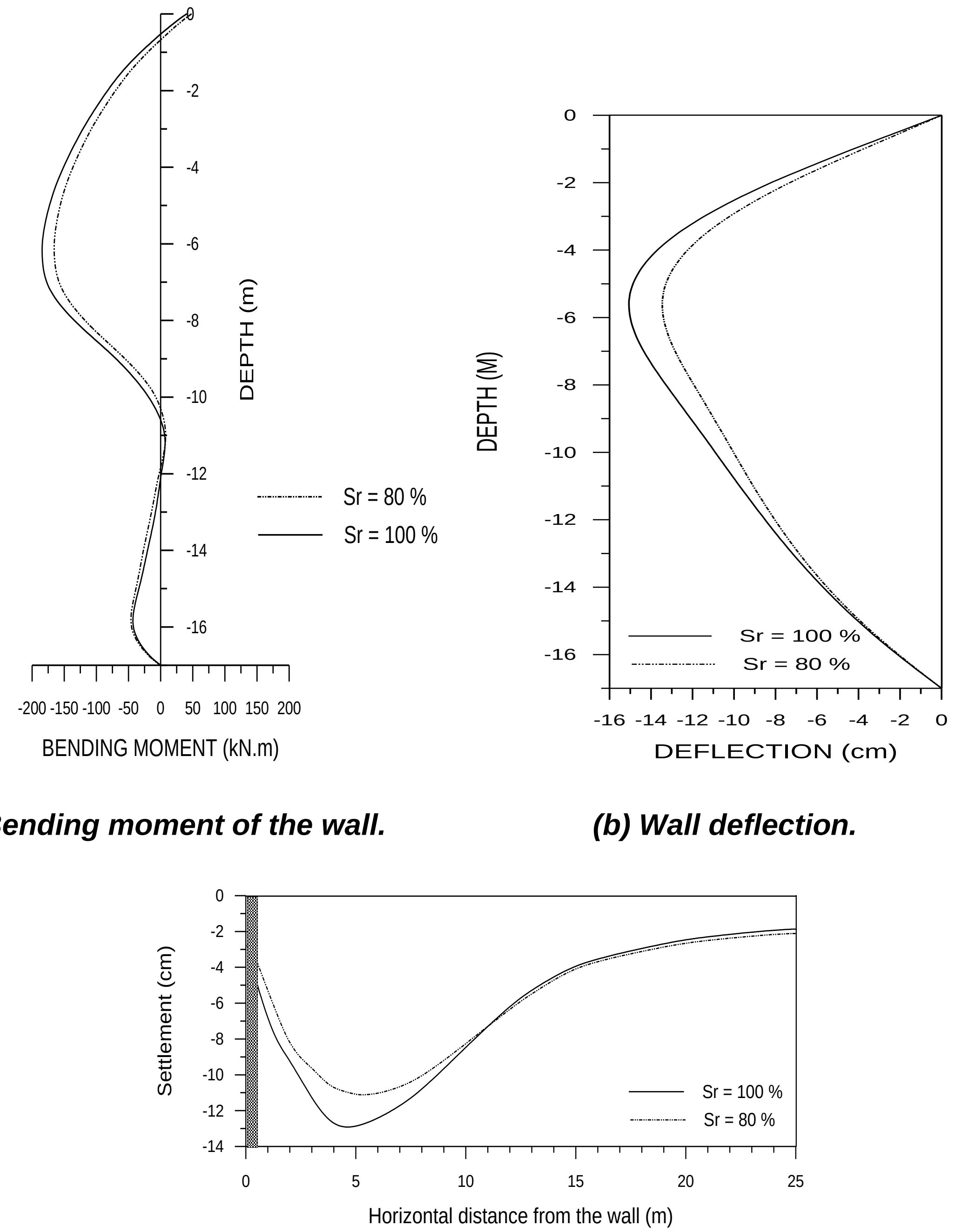
Sr = 80 %

Figure (9): Predicted deformations of the soil at the end of excavation (exaggerated by a factor of 10).

2- Supported excavation with strut.

In this case of supported excavation with the strut, the same dimensions of the problem in the first case are used, but in this case the strut is entered at the top of the wall. The analysis is carried out for the two states of soil fully saturated and partially saturated soil with degree of saturation equal to (80%). From figures (10 a and b), it can be seen that the difference between the fully and partially saturated soil is small because the strut prevents the soil movement at the surface. This leads to dissipate the effect of partially saturated soil.

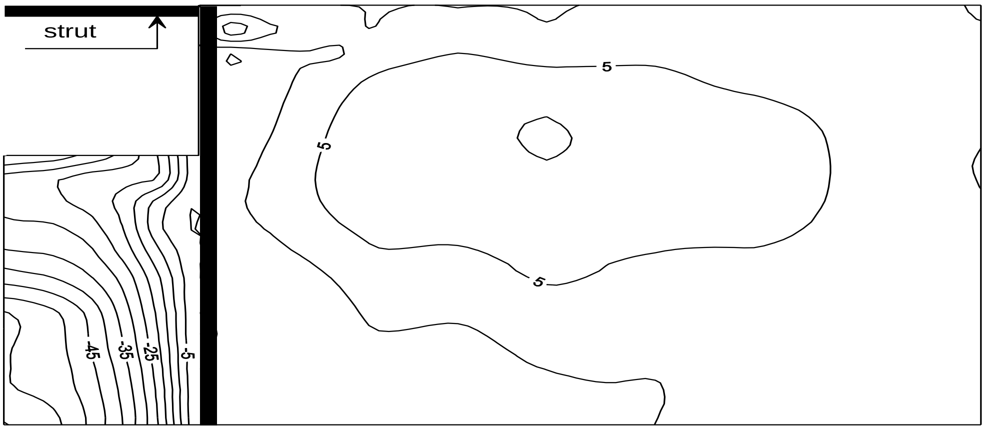
The shear stresses and excess pore water pressure are shown in Figures. (11) and (12), respectively. From these two figures, it can be seen that the shear stresses in the state of partially saturated soil are the greater and the excess pore water pressure is the smallest because of the effect of air within the soil.



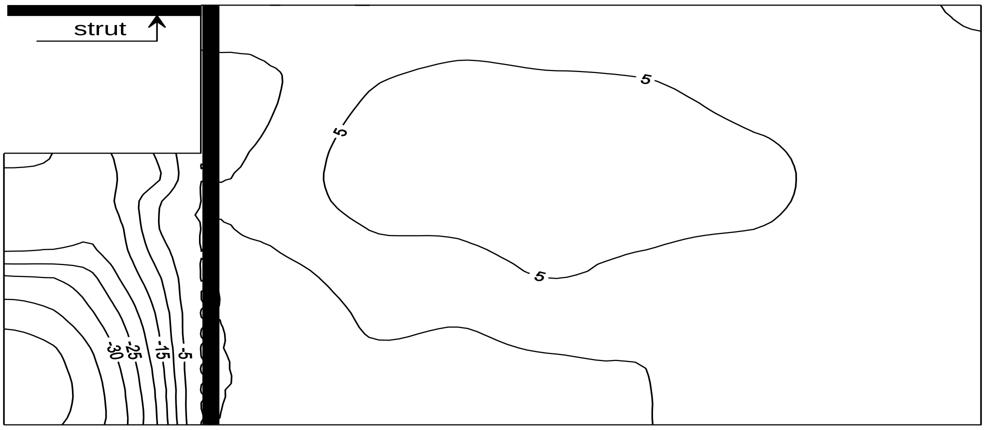
(a) Bending moment of the wall. ***(b) Wall deflection.***

(c) Surface movement of the soil.

Figure (10): Predicted deformations and bending moment for supported excavation with strut.



a): Sr = 100 %



b): Sr = 80 %

Figure (11): Contours of excess pore water pressure (kPa) in the case of supported excavation with strut.

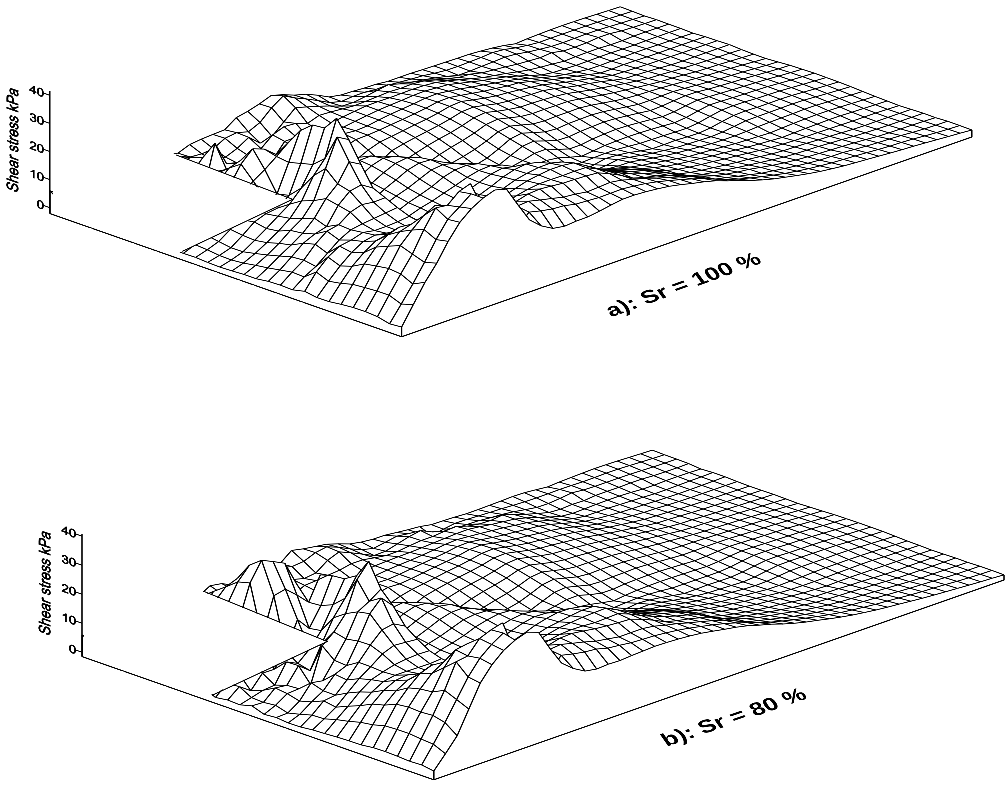


Figure (12): Predicted shear stresses in the soil in the case of supported excavation with strut at the end of excavation.

6. Supported Excavation in Soft Clay

In this section, the field results of a real supported excavation in fully and partially saturated soft clay constructed in Norway and reported by Clausen (1971) which used linear elastic model of soil, is analyzed by the developed program (EXCCONPSS) considering that the soil is fully and partially saturated obeying nonlinear modified Cam – Clay Model. The sequences of excavation and the interface behavior between the soil and retaining structures were included in the analysis. Comparisons of the predicted finite element results with field measurements at various excavation stages are presented.The soil within the area of 11.0x11.0m is excavated in eight stages simulated, as shown in Table (2). Figure (13) shows the excavation cross section, subsoil profile, excavation sequence and boundary conditions (Clausen (1971)). The finite element idealization employed in the analyses, with the excavation sequences and boundary conditions which are shown in Figure (14), consists of 245 elements and 741 nodal points. Plane strain eight-noded isoparametric elements are used for discretizing the soil, and isoparametric bar elements with three nodes are utilized for the wall and strut members.The behavior of soil is treated as a nonlinear elasto-plastic material. The interface is assumed to be linear elastic with slip mode defined by the Mohr - Coulomb criterion .The wall and struts are assumed to be linearly elastic materials. The material properties for the soil, interface, wall, and struts are, as listed in Table (3). An iterative solution technique taking into account the changes in conditions during various stages of excavation and in stresses developed in the soil during the installation of the struts is employed.

Table (2): Sequences of real supported excavation in soft clay.

STAGE	DETAIL
1	In-situ stresses, install wall, excavate to -2.8 m.
2	Install strut A, excavate to -5.0 m.
3	Install strut B, excavate to -6.0 m.
4	Install strut C, excavate to -7.0 m.
5	Excavate to -8.0 m.
6	Install strut D, excavate to -9.0 m.
7	Excavate to -10.0 m.
8	Install strut E, excavate to -11.0 m.

Wall properties: E = 2.76E+7 kN/m^2, $\nu = 0.33$

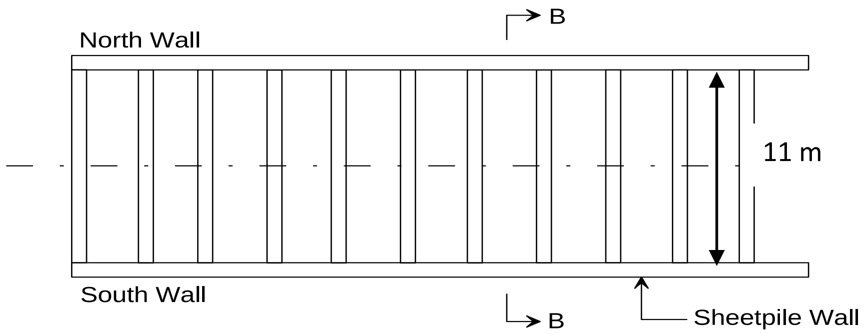
Interface properties: E = 4000 kN/m^2, $\nu = 0.33$, G = 1.0 kN/m^2, t = 0.05 m

Strut properties: A and D: E = 9120 kN/m^2, Area = 1.0 m^2

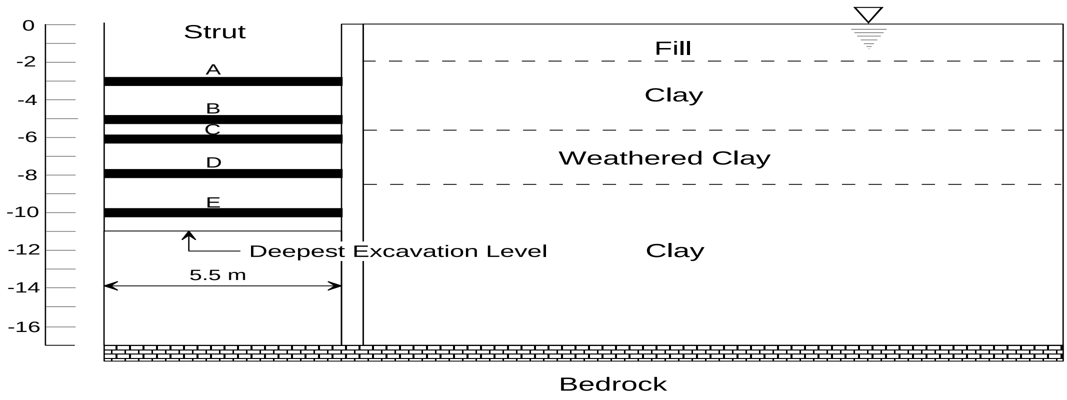
B, C and E: E = 45600 kN/m^2, Area = 1.0 m^2

Table (3): Material properties of real supported excavation (Clausen (1971))

Soil Number	Soil Properties										
	E kN/m^2	ν	γ kN/m^3	ϕ'	K_o	k m/day	S_r %	λ	κ	Γ	M
1	8000	0.4933	19.5	20.5	0.65	8E-5	100	0.161	0.062	2.759	0.8
2	7600	0.4933	19.5	20.5	0.65	8E-5	100	0.161	0.062	2.759	0.8
3	9000	0.4933	19.5	20.5	0.65	8E-5	100	0.161	0.062	2.759	0.8
4	5000	0.4967	19.5	20.5	0.65	8E-5	100	0.161	0.062	2.759	0.8
5	4100	0.4967	19.5	20.5	0.65	8E-5	100	0.161	0.062	2.759	0.8
6	3680	0.30	19.5	20.5	0.65	8E-5	70	0.161	0.062	2.759	0.8



(a) Plane View



(b) Section B-B

Figure (13): Plane view and detail of real supported excavation in soft clay.

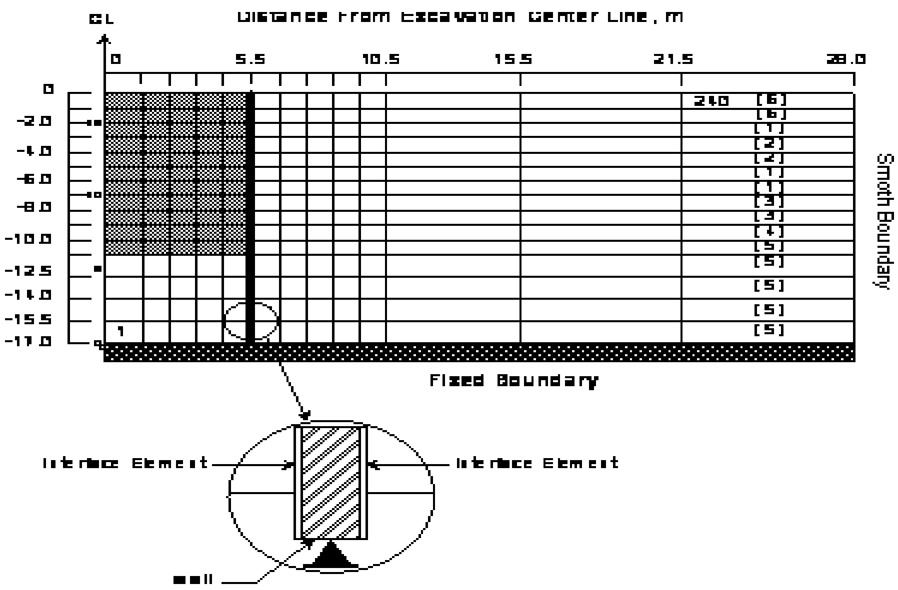


Figure (14): Finite element mesh for real supported excavation in soft clay.

7. Comparison Between Field and Numerical Results

As stated previously, the analyses are carried out using the present method to simulate multi-stage excavation with modified Cam clay model to represent the behavior of fully saturated soil and interface element between soil and wall. Thus, the results presented herein include comparisons of stresses and deformations for various stages of the construction sequences. Figures(15a to d) show comparisons between the computed horizontal stresses (wall pressures) the neighboring soil elements and the observed wall pressures at the end of two, three, five, and seven stages of the construction sequences. It can be seen, that the computed pressures show approximately similar behavior and give good agreement with the field observations especially in active pressures.

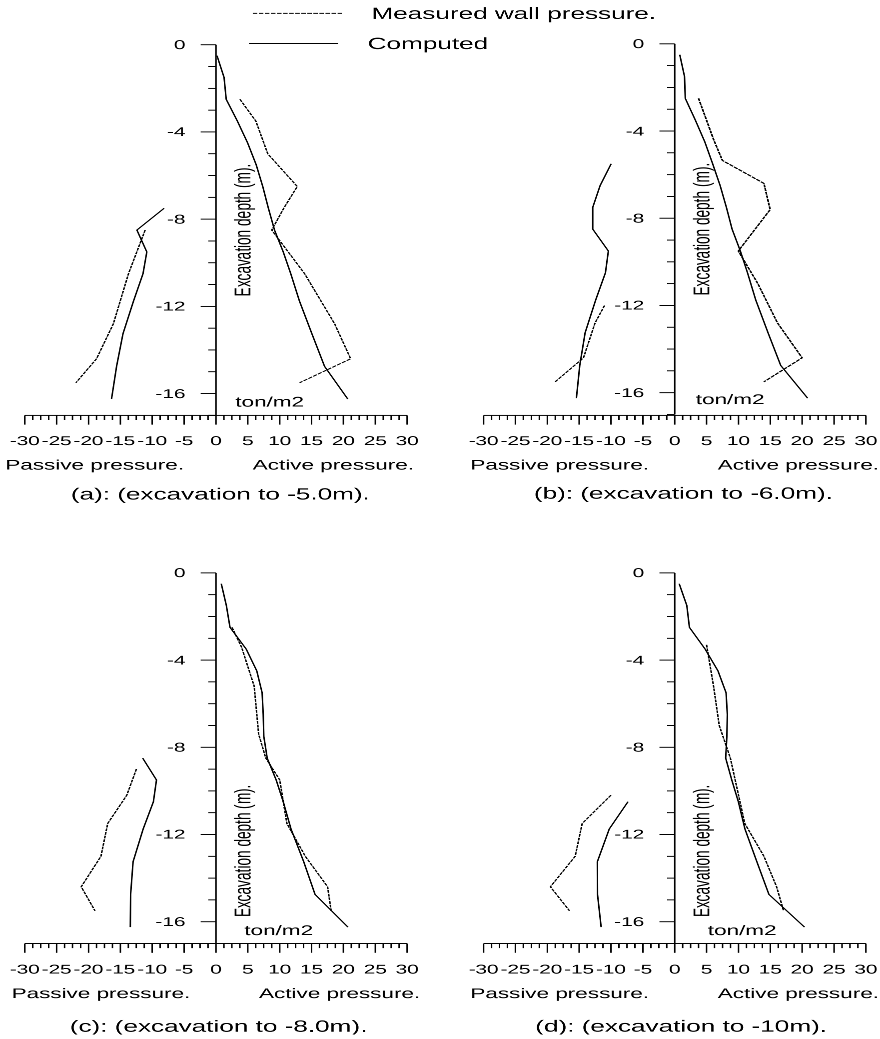
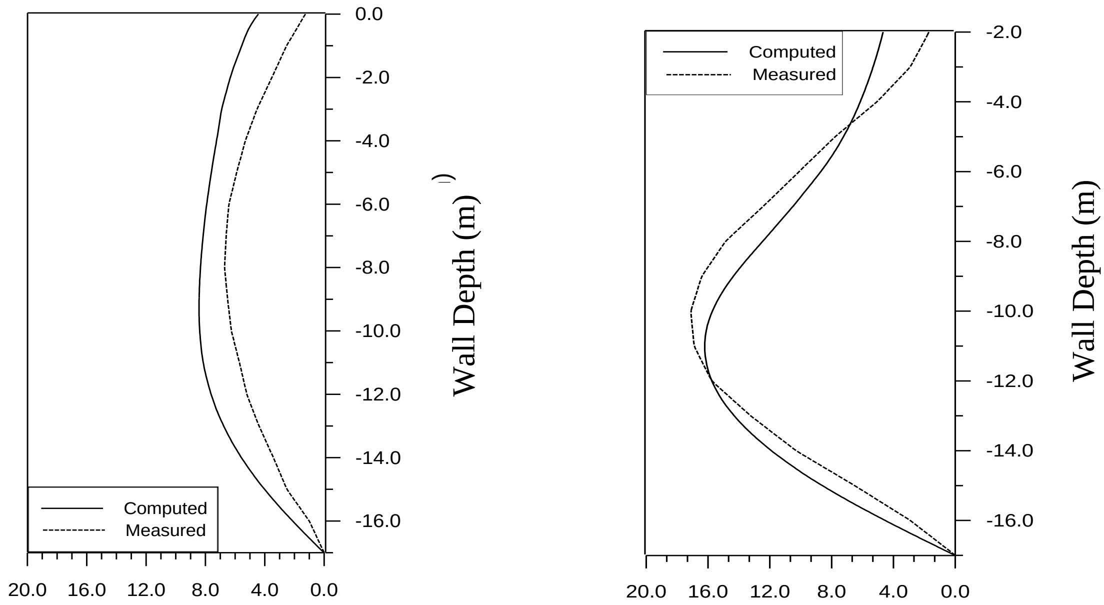


Figure (15): Comparisons of predicted nonlinear analysis and observed wall pressures.

Figures (16 a and b) show comparisons between two predictions, wall deflection with field observations at the end of stages two and seven, respectively. From these figures, a small difference between measured and predicted values can be seen and this difference does not exceed (2 cm). Figures (17 a and b) show vertical surface movement at two stages.

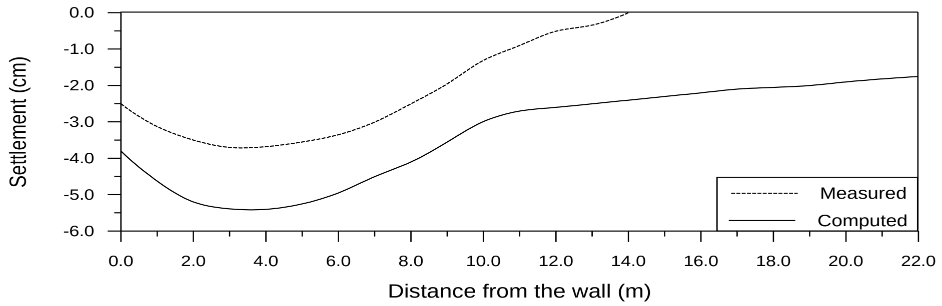
Figure (18) shows predicted ranges of maximum wall deflections compared with field observations. These results are predicted by analyzing the problem using the present method to simulate excavation and modified Cam clay to simulate soil behavior.



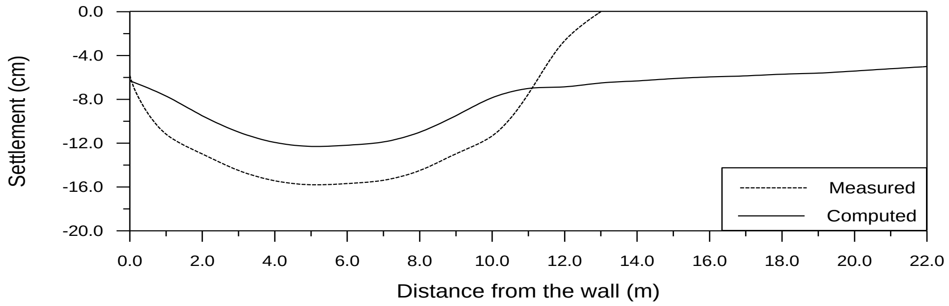
a): (excavation to -5.0m) b): (excavation to -10 m)

Lateral wall movement (cm)

Figure (16): Comparisons of predicted wall deflection with observed values.



(excavation to -5.0 m)



(excavation to - 10.0 m)

Figure (17): Comparisons of predicted surface movement with observed values.

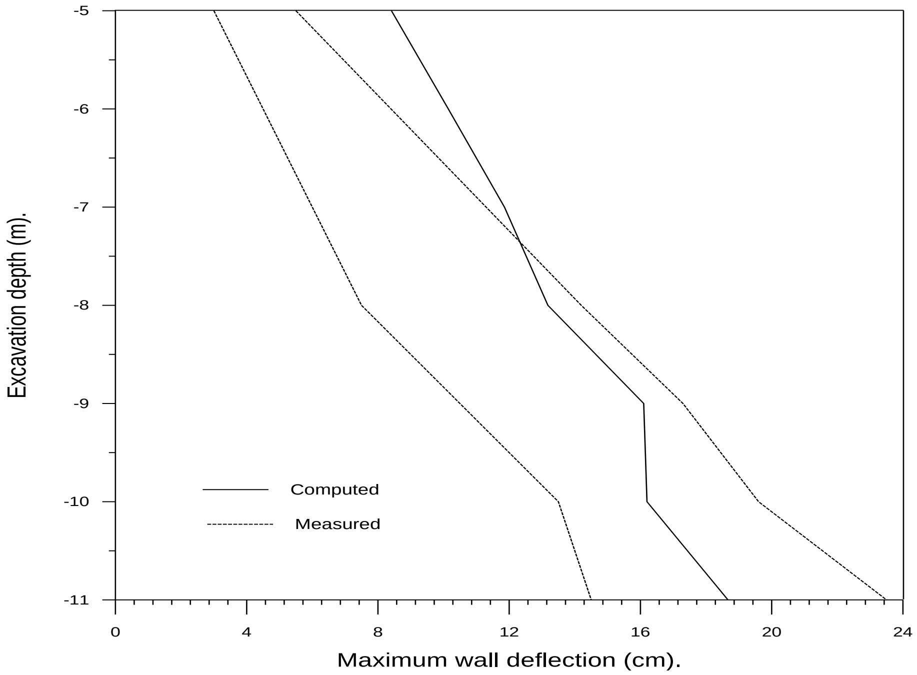


Figure (18): Comparisons of predicted maximum wall deflection with range of measured values.

8. Summary and Conclusions

This paper is directed towards studying the effect of elasto-plastic soil behavior in problems of supported excavation in different cases of saturation conditions (fully and partially saturated soils). The predicted results obtained in this paper show that the degree of saturation gives a major affect on the deformations and shear stresses in the soil. Also the comparisons between fully and partially saturated soils at degree of saturation 80% prove that the maximum lateral wall movements and the vertical surface movement behind the wall decrease in the case of partially saturated soil. This is due to the fact that in case of fully saturated soil the effective stress is decreases allowing for more deformations. The maximum shear stresses seem to be increased because of the increasing in the friction between the soil particles due to air voids. Analyses of a realistic supported field excavation in soft clay were presented in this paper to verify the results obtaining by using the developed computer program “EXCCONPSS”. These analyses were carried out using the finite element method in modified Cam – Clay soil and including the effect of the interface behavior between the soil and the supporting structure.

Based on these analyses, it is possible to conclude that, the incremental nonlinear finite element method can be powerful in analyzing complex time dependent supported excavations. These analyses are based on an incremental modified Cam – Clay model whose parameters are determined from the results of standard laboratory tests. The results of these analyses proved to be very effective means of investigating complex excavation problems and the finite element method can provide more accurate predictions as compared with field measurements.

9. References

1. Atkinson, J.H. and Bransby, P.L. "**The Mechanics of Soil, an Introduction to Critical State Soil Mechanics**" McGraw-HILL Book Company (UK) Limited, 1978.
2. Desai, C.S., Zaman, M.M., Lightner, J.G., and Siriwardane, T.H.J., "**Thin-Layer Element for Interfaces and Joints**" International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 8, No. 1, 1984, pp.(19-43).
3. Biot, M. "**General Theory of Three-Dimensional Consolidation**" Journal of Applied Physics, Vol. 12, 1941, pp. (155-164).
4. Small, J.C and A.K.L. "**Use of Coupled Finite Element Analysis in Unsaturated Soil Problems**". International Journal for Numerical and Analytical Methods in Geomechanics, vol. 24, 2000, pp. (73-94).
5. Konstantinos "**Development, Implementation and Application of Partially Saturated Soil Models in Finite Element Analysis**" thesis submitted to the University of London in partial fulfillment of the requirements for the degree of Doctor of Philosophy in civil engineering (2003).
6. Britto, A.M., and Gunn, M.J., "**Critical State Soil Mechanics via Finite Elements**" Ellis Horwood Limited, U.K., 1987.
7. Borja, P.T., Lee, S.R., and Seed, R.B., "**Numerical Simulation of Excavation in Elasto-plastic Soils,**" International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 13, 1989, pp. (231-249).
8. Lewis, R.W., Roberts, G.K., and Zienkiewicz, O.C., "**A non-linear flow and deformation analysis of consolidation problems,**" Proceeding Second International Conference in Numerical Methods in Geomechanics, Blacksburg, Virginia, U.S.A., Vol.2, 1976, pp.(1106-1118).
9. Lowe, J. and Johnson, T. C. "**Use of Back Pressure to Increase the Degree of Saturation of Triaxial Test Specimens**" Proc. ASCE Research Conf. on the Shear Strength of Cohesive Soils, Boulder, Colo., 1960. Cited by Ref. (58).
10. Clausen, C.J.F "**Finite Element Analysis of Strutted-excavation at Vaterland 1 "**Report No. 52601-2, Norwegian Geotechnique Institute, Oslo, Norway, 1971.