

## Image Impedance Improvement in T and $\pi$ -Networks

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### Abstract :

*T and  $\pi$  networks find applications in the transmission line representations and in the simple filter design. These networks have the problem of a nonconstant image impedance over all the passband response . In this paper, new structures for T and  $\pi$ -networks are suggested to improve their performance. The new structures employ the concept of m-derived technique. The simulation examples demonstrate the effectiveness of the developed methodologies.*

**Keywords :** *Two-port networks , Image impedance.*

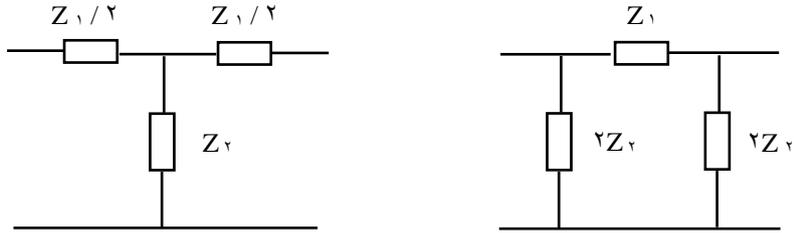
*تحسين الممانعة الصورية لشبكات T,  $\pi$*

*الخلاصة:*

*إن شبكات T و  $\pi$  تستخدم في تطبيقات تمثيل خطوط نقل الإشارة الكهربائية و في تصميم دوائر مرشحات التردد. هذه الشبكات تعاني من مشكلة عدم ثبوت ممانعتها الصورية على كل حزمة الامرار وخصوصا قرب تردد القطع وبالتالي جزء من هذه الحزمة يجب ان تهمل. في هذا البحث تم اقتراح هيكلية جديدة لهذه الشبكات والهدف هو لتحسين ادائها. التصميم الجديد يستغل مبدء m-derived المعروفة. ان نتائج الامثلة التي تم اجرائها توضح فاعلية طرق التصميم المطورة.*

### 1. Introduction:

An ideal filter (or transmission line) is a cascade of linear two-port network in T or  $\pi$ -form that provides perfect transmission of electrical signal for frequencies in a certain passband region, infinite attenuation for frequencies in the stopband region. The goal of filter design is to approximate the ideal requirements and to achieve the ideal response within acceptable tolerance with circuits consisting real components (R, L, and C). the T and  $\pi$ -circuits are illustrated in Fig. 1 .



**Fig. 1 Two-port networks. (a) T-form. (b) π-form**

The image impedance ( $Z_i$ ) is defined as the impedance looking into the input (or output) port when the output (or input) port is also terminated in  $Z_i$ . For T and  $\pi$ -networks, the image impedances are given by [1]

$$Z_{iT} = \sqrt{Z_1 Z_2} \sqrt{1 + \frac{Z_1}{4Z_2}} \dots\dots\dots (1)$$

$$Z_{i\pi} = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1 + \frac{Z_1}{4Z_2}}} \dots\dots\dots (2)$$

The relationships between these impedances are

$$Z_{iT} = \frac{Z_1 Z_2}{Z_{i\pi}} \dots\dots\dots (3)$$

$$Z_{i\pi} = \frac{Z_1 Z_2}{Z_{iT}} \dots\dots\dots (4)$$

By using real elements instead of  $Z_1$  and  $Z_2$ , the LPF and HPF circuits will be achieved as shown in Fig. 2 [7].

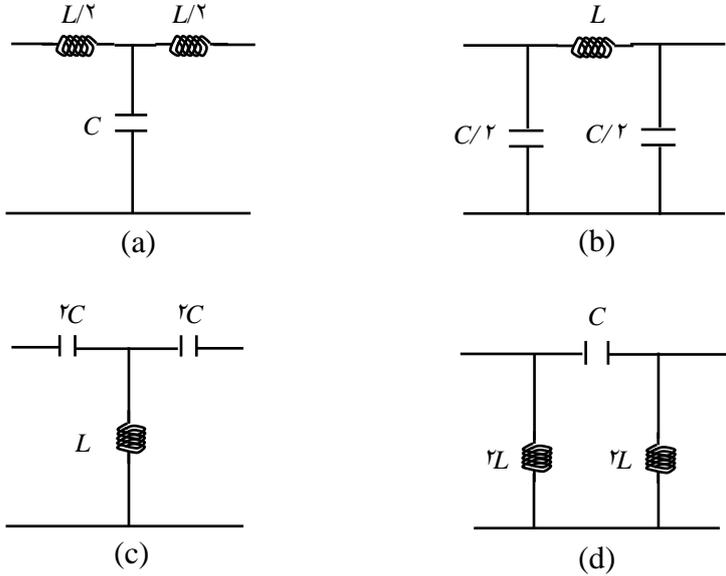


Fig. 2 Filter circuits. (a) T-LPF. (b)  $\pi$ -LPF. (c) T-HPF. (d)  $\pi$ -HPF.

The image impedance of the four circuits can be found as

$$(Z_{iT})_{LPF} = Z_o \sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2} \dots\dots\dots(5)$$

$$(Z_{i\pi})_{LPF} = \frac{Z_o}{\sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}} \dots\dots\dots(6)$$

$$(Z_{iT})_{HPF} = Z_o \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2} \dots\dots\dots(7)$$

$$(Z_{i\pi})_{HPF} = \frac{Z_o}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} \dots\dots\dots(8)$$

where the cutoff frequency  $\omega_c$  for the LPF is defined as

$$\omega_c = \frac{2}{\sqrt{LC}} \dots\dots\dots(9)$$

and for the HPF as

$$\omega_c = \frac{1}{2\sqrt{LC}} \dots\dots\dots(10)$$

and the characteristic impedance is

$$Z_o = \sqrt{\frac{L}{C}} \dots\dots\dots(11)$$

To gain perfect impedance matching and maximum power transferring, it is necessary to approximate the ideal response of the filters. This condition is achieved when the filter sections have the same characteristic impedance and the overall filter circuit is terminated in its image impedance at both input and output ports. This is a major weakness of the design, because the image impedance is a function of frequency (see equations 9 to 11) which is not likely to match a given source or load impedance  $Z_o$  specially when  $\omega$  is nearing  $\omega_c$ . As a solution to this problem, all the previous works used the T-networks with two buffer stages called m-derived half sections at the ports of the filter as shown in Fig. 3 [3] to [5].

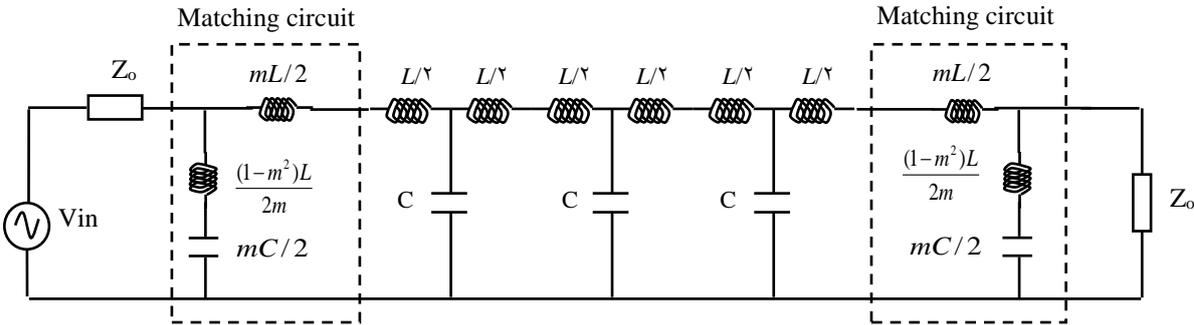


Fig. 3 M-derived terminating buffers with T-LPF.

A feature of an m-derived half section is that it exhibits identical image impedance as the T-filter. It is given as [6]

$$Z_i = Z_o \frac{1 - [(1 - m^2)\omega^2 / \omega_c^2]}{\sqrt{1 - (\frac{\omega}{\omega_c})^2}} \dots\dots\dots(12)$$

This matching section provides fairly constant impedance up to frequencies near the cutoff frequency. It has been shown that  $m=0.7$  gives the optimal results [9]. This technique increases the complexity of the circuit design and optimization, also there is no such matching section for the  $\pi$ -networks yet.

The problem of nonconstant image impedance can also be overcome with the following proposed methodologies.

**2. Proposed Structures for T and  $\pi$ -Networks:**

The idea of inserting a factor  $m$  in the impedance equation may also be used to modify the image impedance of the two-port network itself. We begin with the  $\pi$ -network. As shown in Fig. 4a,b the impedances  $Z_1$  and  $Z_2$  are replaced with  $Z_1'$  and  $Z_2'$ , and we let

$$Z_2' = mZ_2 \dots\dots\dots(13)$$

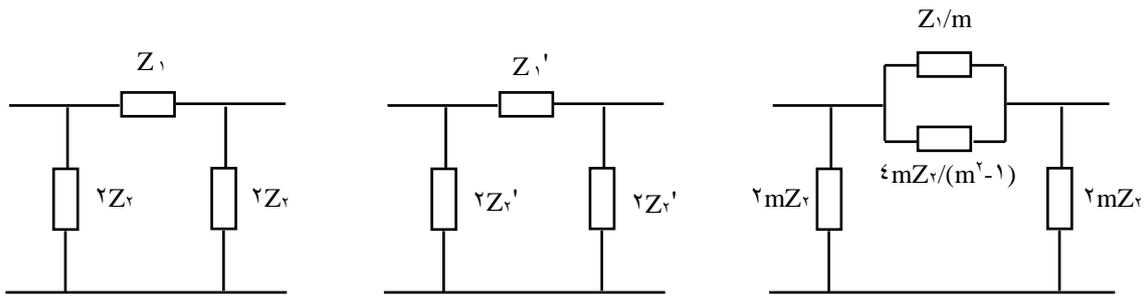
then we choose  $Z_1'$  to obtain the same image impedance value of the constant- $\pi$  section. Thus

$$Z_{i\pi} = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1 + \frac{Z_1}{4Z_2}}} = \frac{\sqrt{Z_1' Z_2'}}{\sqrt{1 + \frac{Z_1'}{4Z_2'}}} \dots\dots\dots(14)$$

solving for  $Z_1'$  gives

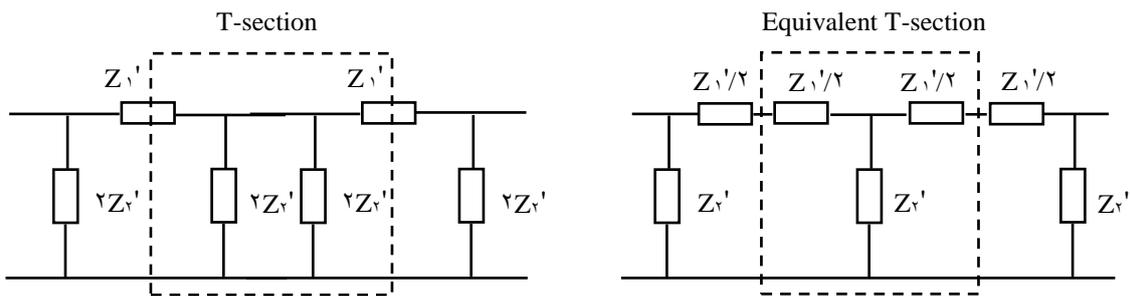
$$Z_1' = \frac{1}{\frac{m}{Z_1} + \frac{m^2 - 1}{4mZ_2}} \dots\dots\dots(15)$$

which means that  $Z_1'$  represents two elements in parallel, as indicated in Fig. 4c .



**Fig. 4 Development of a  $\pi$ -network. (a) Classical  $\pi$ -network. (b) and (c) Proposed  $\pi$ -network**

Since the modified network have image impedance identical to that of the old  $\pi$ -network, so we still have the problem of a nonconstant image impedance. Now if we consider the  $m$ -derived  $\pi$ -section is a piece of an infinite cascade of such circuits as shown in **Fig. 5**, then we will obtain the corresponding  $m$ -derived T-networks.



**Fig. 5 Development of a T-section. (a) Infinite cascade of  $\pi$ -sections. (b) The embed T-equivalent**

The image impedance of the T-equivalent network is

$$Z_{iT'} = \frac{Z_1' Z_2'}{Z_{i\pi}} = \frac{\sqrt{Z_1' Z_2'} \sqrt{1 + \frac{Z_1'}{4Z_2'}}}{1 + \frac{Z_1'}{4Z_2'} - \frac{Z_1'}{4m^2 Z_2'}} \dots \dots \dots (16)$$

For a LPF, we have  $Z_1 = j\omega L$  and  $Z_2 = 1/j\omega C$ . Then

$$(Z_{iT}')_{LPF} = Z_o \frac{\sqrt{1 - (\frac{\omega}{\omega_c})^2}}{1 - (1 - \frac{1}{m^2})(\frac{\omega}{\omega_c})^2} \dots\dots\dots(17)$$

and for a HPF, we have  $Z_1 = 1/j\omega C$  and  $Z_2 = j\omega L$ , then

$$(Z_{iT}')_{HPF} = Z_o \frac{\sqrt{1 - (\frac{\omega_c}{\omega})^2}}{1 - (1 - \frac{1}{m^2})(\frac{\omega_c}{\omega})^2} \dots\dots\dots(18)$$

The impedance became a function of m, so we can choose m to minimize its variation over the passband of the filter. Many values were tested, we found that a value of  $m = 1,66$  generally gives the best results.

For T-networks, the equivalent impedance for LPF and HPF are found same way. They are

$$(Z_{i\pi}')_{LPF} = Z_o \frac{1 - (1 - m^2)(\frac{\omega}{\omega_c})^2}{\sqrt{1 - (\frac{\omega}{\omega_c})^2}} \dots\dots\dots(19)$$

$$(Z_{i\pi}')_{HPF} = Z_o \frac{1 - (1 - m^2)(\frac{\omega_c}{\omega})^2}{\sqrt{1 - (\frac{\omega_c}{\omega})^2}} \dots\dots\dots(20)$$

and the selected value is  $m = 1,66$  for this case.

Fig. 6 summarizes the equivalent m-derived circuits to the classical two-port networks.

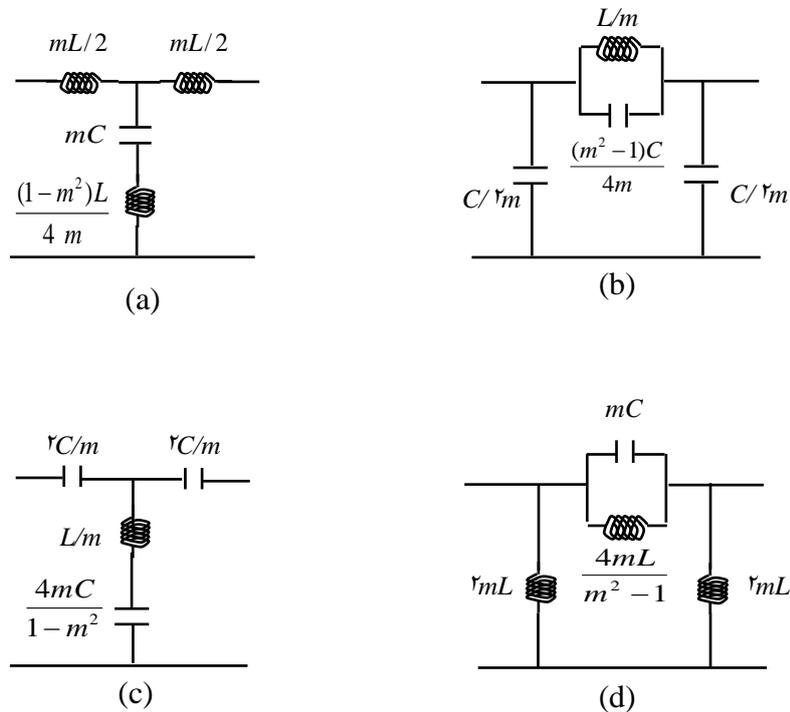


Fig. 6 Proposed m-derived filter circuits. (a)Equivalent T-LPF. (b)Equivalent  $\pi$ -LPF.(c)Equivalent T-HPF.(d)Equivalent  $\pi$ -HPF

### 3. Samples 3-stage Design:

Based on the new topologies of the two-port network, we have designed four filter circuits. The first circuit is a cascade of 3-stage m-derived T-LPF sections, the second is a cascade of 3-stage m-derived  $\pi$ -LPF sections, the third is a cascade of 3-stage m-derived T-HPF, and the fourth is a cascade of 3-stage m-derived  $\pi$ -HPF sections. The cutoff frequency and the characteristic impedance in the four circuits are 1 MHz and  $50 \Omega$  respectively. The m-value is 0.6 for T-networks and 1.66 for  $\pi$ -networks. No matching networks are used between the circuits and their load and source impedances. The source and load impedances are assumed to be  $50 \Omega$ . In order to prove the advantages of the proposed networks, the performance of each of the designed circuit compared with its performance obtained from the classical structure. The simulated gain responses of the circuits are shown in figures from 7 to 10. It is clear that there are large degradation in the responses of the circuits of the classical two-port networks specially near the cutoff frequency. These degradations are remedied by using the proposed topologies of the two-port sections in the filter structures.

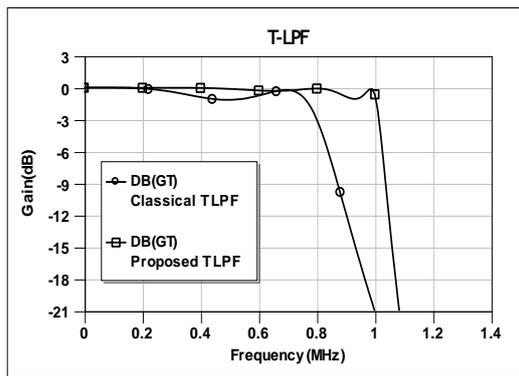


Fig. 7 Gain response of 3-stage T-LPFs

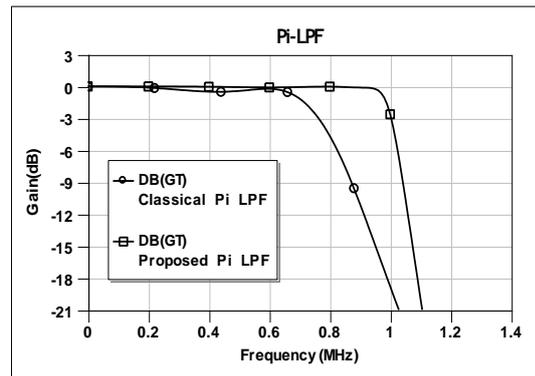


Fig. 8 Gain response of 3-stage  $\pi$ -LPFs

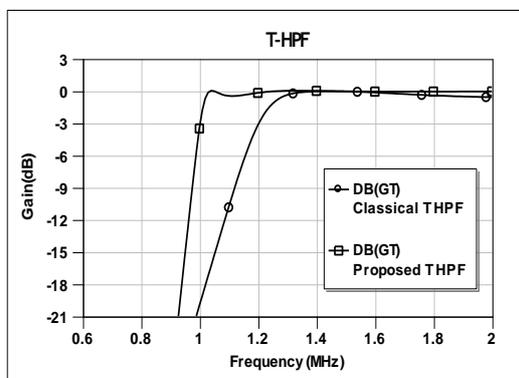


Fig. 9 Gain response of 3-stage T-HPFs

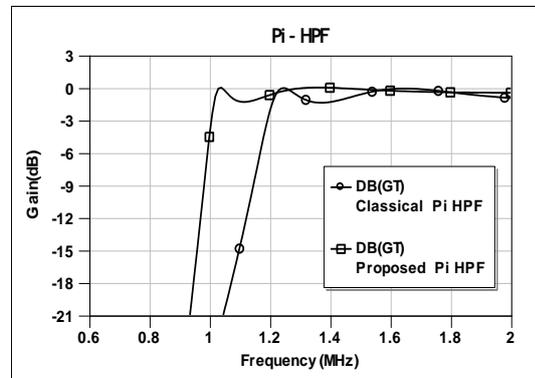


Fig. 10 Gain response of 3-stage  $\pi$ -HPFs

#### 4. Conclusion:

In this paper, we have proposed new structures for the two-port network. The goal was to improve the image impedance of the T and  $\pi$ -circuits, thereby the gain degradation in their frequency response can be corrected.

The comparison of the performance of the new T and  $\pi$ -circuits to the classical circuits show the effectiveness of the proposed design methodologies. Another advantage was achieved, that is the matching circuits were not necessary to be used.

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