Torsional Behavior of
High-Strength Reinforced Concrete Beams

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ABSTRACT:

Recent methods for torsional design of reinforced concrete beams tend to the use of space truss analogy, instead of the earlier skew bending theory. A total of (٤٣) rectangular beams made of high strength concrete (HSC) that failed under pure torsion are considered in this work. These have been taken from the literature.

Regression analysis was performed on the results to obtain two representative equations to predict: cracking torsional moment T_{cr} and torsional resistance moment T_r. The first equation is based on (٣) major parameters that include concrete compressive strength f'_c and sectional dimensions, while the second one is based on (٧) major parameters which include the quantification of the influence of both transverse and longitudinal reinforcement.

When the ACI ٣١٨M-٠٥ Code design equation was applied, it gave a coefficient of variation (COV) of (٤٢٫٨) percent for the ratio of tested / calculated torsional strength ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$), however, the proposed equation has led to a COV of (١١٫٢) percent.

Keywords: *beams; cracking torsional moment; high strength concrete; longitudinal reinforcement; torsional resistance moment; transverse reinforcement.*

سـلوك اللـي فـي العتبـات الخرسـانية المسلحـة
المصنوعـة من خرسـانة عاليـة المقاومـة

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الخـلاصـة:

تعتمد الطرق الحديثة لتصميم اللي فـي العتبات الخرسانية المسلحة علـى نظرية المسنم الفضائي (*Space Truss*) بدلا من الإنشاء المائل (*Skew Bending*). تمت دراسة (٤٣) عتبة خرسانية مسلحة مستطيلة المقطع مصنوعة من خرسانة عالية المقاومة فشلت تحت تأثير اللي الخالص مأخوذة من بحوث سابقة.

حللت النتائج بطريقة التحليل الإرتدادي للحصول على معادلتين لحساب عزم اللي الذي يسبب التشقق (Tcr)
ومقاومة اللي التصميمية (Tr) . تعتمد المعادلة الأولى على ثلاثة معاملات رئيسية هي مقاومة إنضغاط الخرسانة وأبعاد
المقطع، بينما تعتمد المعادلة الثانية على سبعة معاملات رئيسية تشتمل على التقييم الكمي لتأثير حديد التسليح الطولي
والعرضي للعتبات.

عندما طبقت طريقة التصميم للمدونة (ACI ٣١٨M- ٠٥)، تم الحصول على معامل تغاير مقداره (%٤٢٫٨)
لنسبة مقاومة اللي العملية / المقاومة التصميمية (Tu-test / Tr-calc). وبالمقارنة فأن تطبيق المعادلة المقترحة نتج عنه
معامل تغاير مقداره (%١١٫٢)

INTRODUCTION:

Use of HSC leads to more economical building structures resulting from small sections of structural members and large usable floor areas. It also leads to a reduction in overall building height and dead loads resulting from the use of thinner slabs and shallower beams [١,٢,٣,٤].

Pure torsion only occurs infrequently in practice. Normally, it arises as a combined action with bending and/or shear. It can become a predominant action in structures such as eccentrically loaded box beams, curved girders, spandrel beams, structures of irregular shapes, and spiral staircases [٥,٦]. However, in bridges, torsion constitutes a significant design action because of eccentric forces. Since large bridge construction is an obvious application of HSC, an investigation of reinforced HSC beams subjected to pure torsion is of interest. In ١٩٦٩ the then ACI Committee ٤٣٨ published its report recommending torsional design based on the skew bending theory [٧]. The ACI ٣١٨ Committee used this theory starting from the ١٩٧١ Code [٨] which continued up to the ١٩٨٩ Code [٩]. BS-٨٥ [١٠] and BS-٩٧ [١١] Code versions also used the same approach.

The most recognized theoretical model of pure torsion in reinforced concrete is the space truss model. Based on post-doctoral research published by MacGregor and Ghoneim[١٢], the ACI Code in ١٩٩٥[١٣] accepted this model. This is now included in the latest ACI ٣١٨-٠٨ Code [١٤]. The Canadian [١٥], AASHTO-LRFD [١٦], and European [١٧] Codes also use space truss analogy for torsional design.

There are a number of more accurate but more complex design procedures in the literature, [١٨,١٩] but they are not considered in this work.

RESEARCH SIGNIFICANCE:

This paper provides an evaluation of the design provisions for pure torsion based on (٧) different code approaches: ٢ using skew bending theory (ACI ٣١٨M-٨٩[٩] and BS-٩٧[١١]); plus ٥ using space truss analogy (ACI ٣١٨M-٩٩[٢٠], ACI ٣١٨M-٠٥[٢١], Canadian [١٥], AASHTO-LRFD [١٦], and EURO [١٧]). In addition, a number of equations adopted by some researchers to predict T_{cr} value are included. A total of (٤٣) tests of torsional failure of tested beams is used to evaluate the previous (٧) methods. Two proposed equations which are based on regression analysis are also introduced. The first one estimates the cracking torsional

moment T_{cr} of HSC beams, while the second one predicts the torsional resistance moment of such beams.

EXPERIMENTAL RESULTS:

All available experimental results from test series on pure torsion are obtained from the literature. The ranges of the variables of these (٤٣) rectangular solid section beams are listed in Table (١). The main significant parameters are concrete compressive strength f'_c, aspect ratio $\left(\frac{y}{x}\right)$, sectional area A_{cp}, nominal stirrup strength $\rho_v.f_{yt}$, and nominal longitudinal steel strength $\rho_\ell.f_{y\ell}$. These beams include ٥, ١٠, ١٢, ٨, and ٨ specimens from the references ٢٢, ٢٣, ٢٤, ٥ and ٢٥ respectively.

Table ١- Ranges of the variables for the ٤٣ tested beams.

Detail	f'_c (MPa)	$\dfrac{y}{x}$	A_{cp} (mm²)	$\rho_v.f_{yt}$ (MPa)	$\rho_\ell.f_{y\ell}$ (MPa)
Low	٤١,٧٠٠	١,٤٣٠	٤٠٠٠٠	١,٧٠٠	٢,٨٨٠
High	١٠٩,٨٠٠	٤,٠٠٠	١٧٥٠٠٠	٧,٢٥٠	٢٤,٧٠٠
High/Low	٢,٦٣٣	٢,٧٩٧	٤,٣٧٥	٤,٢٦٥	٨,٥٧٦

Where:

f'_c = cylinder compressive strength of concrete, MPa

x = the shorter side of the cross section, mm

y = the longer side of the cross section, mm

A_{cp} = area enclosed by outside perimeter of concrete cross section, mm²

ρ_v = stirrups ratio = $\dfrac{2A_t}{b.s}$

f_{yt} = specified yield strength of transverse reinforcement, MPa

ρ_ℓ = longitudinal steel ratio = $\dfrac{A_\ell}{b.h}$

$f_{y\ell}$ = specified yield strength of longitudinal reinforcement, MPa

EVALUATION OF EXPERIMENTAL RESULTS:
Cracking Torsional Moment Equations:

Following are the methods considered in this work to estimate the cracking torsional moment of the beams:

١. ACI ٣١٨M-٨٩ Code [١] method:

$$T_{cr} = \frac{1}{6}\sqrt{f_c'} \cdot \sum x^2 \cdot y \tag{١}$$

Where:

T_{cr} = cracking torsional moment, N.mm

٢. ACI ٣١٨M-٠٥ Code [٢١] method:

$$T_{cr} = 0.33\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{٢}$$

Where:

P_{cp} = outside perimeter of concrete cross section, mm.

٣. Canadian-٩٤ Code [١٥] method:

$$T_{cr} = 0.4\phi_c\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{٣}$$

Where $\phi_c = ٠,٦$

٤. H$_{su}$ and M$_o$'s [١٨] method:

$$T_{cr} = 0.5\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{٤}$$

٥. Koutchoukali and Belarbi's[٥] method:

$$T_{cr} = 0.46\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{٥}$$

٦. Fang and Shiau's[٢٥] method:

$$T_{cr} = 0.095\sqrt{f_c'}\,x^2\,y \tag{٦}$$

Torsion Design Equations:

(٧) Methods of existing design codes are included in this study to predict the torsional resistance moment of the beams. To make comparison between design methods, torsional

resistance $T_{r-calc.}$ is used instead of nominal $T_{n-calc.}$ throughout (e.g. $T_{r-calc.} = ٠,٨٥ \ T_{n-calc.}$ per ACI ٣١٨M-٨٩ Code [٩] method).

The design code methods are based on two approaches:

a. Skew Bending Theory:

Torsional strength of beams is composed of two parts: the concrete contribution T_c and the reinforcement contribution T_s.

١. ACI ٣١٨M-٨٩ Code [٩] method:

$$T_{r-calc} \leq T_{r\,ACI-t-89} = 0.85 \left[\frac{\sqrt{f'_c}}{15} \sum x^2.y + \alpha_t . \frac{A_t.x_1.y_1.f_{yt}}{S} \right] \qquad (٧,١)$$

$$T_{r-calc} \leq T_{r\,ACI-\ell-89} = 0.85 \left[\frac{\sqrt{f'}_c}{15} \sum x^2.y + \alpha_\ell . \frac{A_\ell.x_1.y_1.f_{y\ell}}{P_h} \right] \qquad (٧,٢)$$

Where:

$T_{r\,ACI-t-٨٩}$ = torsional resistance moment provided by concrete and stirrups, calculated by ACI-٨٩ method, N.mm.

α_t = $٠,٦٦+٠,٣٣(y_١/x_١) \leq ١,٥$

A_t = area of one leg of closed stirrup resisting torsion within spacing S, mm٢.

$x_١$ = shorter centre-to-centre dimension of closed rectangular stirrup, mm.

$y_١$ = longer centre-to-centre dimension of closed rectangular stirrup, mm.

S = spacing of transverse torsional reinforcement in direction parallel to longitudinal reinforcement, mm.

$T_{r\,ACI-\ell-٨٩}$ = torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by ACI-٨٩ method, N.mm.

A_ℓ = area of longitudinal reinforcement required for torsion, mm٢.

P_h = perimeter of centerline of outermost closed transverse torsional reinforcement, mm.

٢. BS ٨١١٠-٩٧ Code [٧] method:

$$T_{r-calc} \leq T_{r\,BS-t} = 0.0375\,x^2 \left(y - \frac{x}{3} \right) \sqrt{f'_c} + \frac{1.6\,A_t.x_1.y_1 \left(0.95 f_{yt} \right)}{S} \qquad (٨,١)$$

$$T_{r-calc} \leq T_{r\,BS-\ell} = 0.0375\,x^2 \left(y - \frac{x}{3} \right) \sqrt{f'_c} + \frac{1.6\,A_\ell.x_1.y_1 \left(0.95 f_{y\ell} \right)}{P_h} \qquad (٨,٢)$$

Where it is assumed that $f'_c = ٠,٨ f_{cu}$,

$T_{r\,BS\text{-}t}$ = torsional resistance moment provided by concrete and stirrups, calculated by BS-٩٧ method, N.mm.

$T_{r\,BS\text{-}\ell}$ = torsional resistance moment provided by concrete and longitudinal torsion reinforcement, calculated by BS-٩٧ method, N.mm.

b. Space Truss Analogy:

This new method is considerably simpler to understand and apply than the previous one. It can also be used for prestressed concrete loaded in torsion, a case not covered by the ACI ٣١٨M-٨٩[٩] Code. It assumes that the concrete contribution $T_c=٠$. In this method, the beam cross section is idealized as a tube. After cracking, the tube is idealized as s space truss consisting of closed stirrups, longitudinal bars in the corners, and concrete compression diagonals approximately centered on the stirrups. The diagonals are at an angle θ to the member longitudinal axis.

The most significant difference between the torsion provisions of the ACI Codes and the AASHTO-LRFD [١٦] specifications is the specified value of θ. For non prestressed sections, the ACI Code recommends (٤٥) degrees, while the AASTHO[١٦] provisions permit a value of about (٣٦) degrees (based on the longitudinal strain at mid-span of the section) [٢٦, ٢٧]. The methods adopted this analogy are:

١. ACI ٣١٨M-٩٩ Code[٢٠] method:

$$T_{r-calc} \le T_{r\,ACI-t-99} = 0.85\left[\frac{1.7A_{oh}.A_t.f_{yt}}{S}\right] \tag{٩,١}$$

$$T_{r-calc} \le T_{r\,ACI-\ell-99} = 0.85\left[\frac{1.7A_{oh}.A_\ell.f_{y\ell}}{P_h}\right] \tag{٩,٢}$$

Where:

$T_{r\,ACI\text{-}t\text{-}٩٩}$ = torsional resistance moment provided by stirrups, calculated by ACI-٩٩ method, N.mm.

$T_{r\,ACI\text{-}\ell\text{-}٩٩}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-٩٩ method, N.mm.

A_o = area enclosed by centerline of outermost closed transverse torsional reinforcement, mm².

٢. ACI ٣١٨M-٠٥ Code [٢١] method:

$$T_{r-calc} \leq T_{r\,ACI-t-05} = 0.75 \left[\frac{1.7 A_{oh}.A_t.f_{yt}}{S} \right] \tag{10.1}$$

$$T_{r-calc} \leq T_{r\,ACI-\ell-05} = 0.75 \left[\frac{1.7 A_{oh}.A_\ell.f_{y\ell}}{P_h} \right] \tag{10.2}$$

Where:

$T_{r\,ACI-t-05}$ = torsional resistance moment provided by stirrups, calculated by ACI-05 method, N.mm.

$T_{r\,ACI-\ell-05}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by ACI-05 method, N.mm.

3. Canadian-94 Code [15] method:

$$T_{r-calc} \leq T_{r\,Can.-t} = 0.85 \left[\frac{1.7 A_{oh}.A_t.f_{yt}}{S} \right] \tag{11.1}$$

$$T_{r-calc} \leq T_{r\,Can.-\ell} = 0.85 \left[\frac{1.7 A_{oh}.A_\ell.f_{y\ell}}{P_h} \right] \tag{11.2}$$

Where:

$T_{r\,Can.-t}$ = torsional resistance moment provided by stirrups, calculated by Canadian Code method, N.mm.

$T_{r\,Can.-\ell}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by Canadian Code method, N.mm.

It can be seen that the Canadian Code [15] method is symmetric with the ACI 318M-99 Code [20] method.

4. AASHTO-LRFD-98 Bridge Design Specifications [11] method:

$$T_{r-calc} \leq T_{r\,AASHTO-t} = 0.85 \left[\frac{1.7 A_{oh}.A_t.f_{yt}}{S}.Cot\,\theta \right] \tag{12.1}$$

$$T_{r-calc} \leq T_{r\,AASHTO-\ell} = 0.85 \left[\frac{1.7 A_{oh}.A_\ell.f_{y\ell}}{0.9 P_h}.\tan\theta \right] \tag{12.2}$$

Where:

$T_{r\,AASHTO-t}$ = torsional resistance moment provided by stirrups, calculated by AASHTO method, N.mm.

$T_{r\ AASHTO\text{-}\ell}$ = torsional resistance moment provided by longitudinal torsion reinforcement, calculated by AASHTO method, N.mm.

θ = angle of inclination of compression diagonals to the member longitudinal axis, equal to ۳٦ degrees.

٥. EURO-۸۹ Code [۱۷] method:

$$T_{r\,EU} = 1.7\,A_{oh}\sqrt{\frac{A_t}{S}.f_{yt}.\frac{A_\ell}{P_h}.f_{y\ell}}$$ (۱۳)

Where:

$T_{r\,EU}$ = torsional resistance moment calculated by EURO method, N.mm.

Statistical Evaluation of Existing Methods:

Table (۲) shows the results of the cracking torsional moment of (۲٦) specimens (out of ٤۳ tested beams- not all the values of T_{cr} are included in the references). The comparison between these results and predicted values ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) leads to a range of (۰,۸٤٦-۱,۹۳۱) for the mean of this ratio. It can be seen that the ACI ۳۱۸M-۸۹ Code [۹] method is the one with the greatest amount (all the ۲٦ specimens) of unacceptable predictions-based on the value of ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) < ۱. The lowest ratio for this code is (۰,۷۱۷).

In contrast, the ACI ۳۱۸M-۰٥[۲۱], Canadian[۱٥], and Fang and Shiau[۲٥] methods lead to good predictions with no results of the previous ratio < ۱, but the ACI ۳۱۸M-۰٥ Code[۲۱] method seems to be the best due to the lowest values of low and high of the ratio ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) among the other two methods. The coefficient of variation (COV) gives a good indication as a measure of the relevance of the prediction method for the ratio ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$). It can be seen that the difference in COV values of all methods is very small (ranging between ۹,۳۸-۹,٤۸ percent), therefore this coefficient does not indicate which method is the best.

Table ۲- Statistical analysis of the ratio ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) for ۲٦ tests.

Detail	ACI-۸۹[۹]	ACI-۰٥[۲۱]	Canadian[۱٥]	Reference (۱۸)	Reference (٥)	Reference (۲٥)	Proposed Eq. (۱٤)
$\bar{x}$	۰,۸٤٦	۱,٤۰٥	۱,۹۳۱	۰,۹۲۷	۱,۰۰۸	۱,٤۸٥	۱,۲۲٥
S.D.	۰,۰۸۰	۰,۱۳۲	۰,۱۸۱	۰,۰۸۷	۰,۰۹٥	۰,۱٤۱	۰,۱۰۹
COV, %	۹,٤۷۹	۹,۳۷۹	۹,۳۷۹	۹,۳۷۹	۹,۳۷۹	۹,٤۷۹	۸,۹۲٦
Low	۰,۷۱۷	۱,۱٥٥	۱,٥۸۸	۰,۷٦۲	۰,۸۲۹	۱,۲٥۷	۱,۰۲۸
High	۰,۹۷۳	۱,٦۳۸	۲,۲٥۲	۱,۰۸۱	۱,۱۷٥	۱,۷۰۸	۱,٤۱۹
High/Low	۱,۳٥۸	۱,٤۱۸	۱,٤۱۸	۱,٤۱۸	۱,٤۱۸	۱,۳٥۸	۱,۳۸۱
Number< ۱	۲٦	۰	۰	۲۲	۹	۰	۰

Table (٣) shows the values of the results of the (٤٣) tested beams, compared with the predicted strength ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$). The range of the mean of this ratio is (١,١٤١-١,٨٥٩). Based on the value of ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$) < ١, the EURO [١٧] method leads to unsafe predictions (١٤ specimens). The lowest ratio for this code is (٠,٣٩٤). On the other hand, the ACI ٣١٨M-٠٥ Code [٢١] method is the most conservative of the existing methods with only (٥) results with the previous ratio < ١. From table (٣) it can be seen that the ACI ٣١٨M-٩٩[٢٠], Canadian [١٥], and ACI ٣١٨M-٠٥[٢١] Code methods lead to the least relevant prediction with a high COV of (٤٢,٨٢١) percent for each one of them. From this point of view, the best COV is (٢٤,١٩٢) percent for the ACI ٣١٨M-٨٩ Code [٩] method. The COV values are (٣٣,٨٧٢, ٣٦,١٣٨, and ٤١,٣٥٥) percent for BS [١١], AASHTO [١٦], and EURO [١٧] methods, respectively.

Table ٣- Statistical analysis of the ratio ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$) for ٤٣ tests.

Detail	ACI-٨٩[٩]	BS[١١]	ACI-٩٩[٢٠] and Canadian[١٥]	ACI-٠٥[٢١]	AASHTO[١٦]	EURO[١٧]	Proposed Eq. (١٥)
$\bar{x}$	١,٣٨٨	١,٢٦٤	١.٦٤١	١,٨٥٩	١,٤٧٢	١,١٤١	١,٢٥٠
S.D.	٠,٣٣٦	٠,٤٢٨	٠,٧٠٢	٠,٧٩٦	٠,٥٣٢	٠,٤٧٢	٠,١٤٠
COV, %	٢٤,١٩٢	٣٣,٨٧٢	٤٢,٨٢١	٤٢,٨٢١	٣٦,١٣٨	٤١,٣٥٥	١١,١٧٢
Low	٠,٨٠١	٠,٦٣٣	٠,٧٠٣	٠,٧٩٧	٠,٥١١	٠,٣٩٤	٠,٩٩٩
High	٢,١١٢	٢,١٧٣	٣,٠٣٠	٣,٤٣٣	٢,٣٥٤	٢,٠٣٣	١,٤٨٩
High/Low	٢,٦٣٥	٣,٤٣٠	٤,٣٠٨	٤,٣٠٨	٤,٦٠٨	٥,١٥٨	١,٤٩٠
Number< ١	٧	١٢	٨	٥	١٠	١٤	١

Regression Analysis of Test Results:

Using regression analysis, the (٢٦) and (٤٣) test results of cracking and resistance moment, respectively were analyzed by computer. The aim is to obtain simple and conservative equations to predict cracking torsional moment and torsional resistance moment

of HSC rectangular section beams under pure torsion, that give the lowest possible COV values of the ratios ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) and ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$). This has led to the following prediction equations:

$$T_{cr-\text{Pr} oposed}=0.115\left(f_c'\right)^{0.6}.x^{1.92}.y \tag{١٤}$$

Where:

$T_{cr\text{-}proposed}$ = cracking torsional moment calculated by proposed method, N.mm

$$T_{r-proposed}=6.2\frac{A_{0h}^{1.2}}{S^{0.55}}\left(\frac{A_t.A_\ell}{P_h}\right)^{0.26}\left(f_{yt}.f_{y\ell}\right)^{0.026} \tag{١٥}$$

Where:

$T_{r\text{-}proposed}$ = torsional resistance moment provided by stirrups and longitudinal torsion reinforcement, calculated by proposed method, N.mm.

Equation (١٤) is based on the ٣ main parameters f'_c, x, and y, while equation (١٥) is based on the ٧ main parameters A_{oh}, S, A_t, A_ℓ, P_h, f_{yt}, and $f_{y\ell}$. Tables ٢ and ٣ had show the summary of statistical evaluation of the proposed methods. The proposed equation (١٤) which estimates T_{cr} gives the best COV value of (٨,٩٢٦) percent among all other methods with no result having the ratio of ($T_{cr\text{-}test}$ / $T_{cr\text{-}calc.}$) < ١ (Table ٢).

As shown in Table (٣), when the proposed equation (١٥) [that predicts T_r) was applied, it led to much safer prediction with only one specimen (out of ٤٣) having the ratio of ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$) < ١ essentially ٠,٩٩٩ $\cong$ ١. It can be seen that there is a great reduction in the COV value that was obtained by applying the proposed equation (١٥) [COV = ١١,١٧ percent]. In addition, the value of high/low of the previous ratio was (١,٤٩) for this equation, while the range of this ratio was (٢,٦٣٥-٥,١٥٨) for all other methods.

To illustrate the relevance of the proposed method – equation (١٥), the ratio of ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$) has been compared by this method with that of the latest available ACI ٣١٨M-٠٥ Code [٢١] procedure (which is the same as the procedure of the ACI ٣١٨-٠٨ Code [١٤]. These are shown in Figs. ١،٢،٣،٤, and ٥.

The comparison in Fig.١ between the ACI ٣١٨M-٠٥[٢١] method and the proposed equation (١٥) shows clearly that for the range of f'_c (٤١,٧ – ١٠٩,٨) MPa, the proposed method shows much less scatter in the results. In addition, the number of unsafe results ($T_{u\text{-}test}$ / $T_{r\text{-}calc.}$) < ١ is greater for the ACI ٣١٨M-٠٥ Code [٢١] method, despite the fact that this ratio is high in several cases (up to ٣,٤٣٣). It is to be noted that there is a tendency toward greater safety with rising f'_c values for both methods which is an important advantage since much fewer tests are made on HSC beams in torsion.

Similar conclusions regarding the much greater scatter and the number of unsafe results by the ACI ٣١٨M-٠٥ Code [٢١] method can be seen in Fig.٢ (influence of the aspect

ratio $\frac{y}{x}$), ٣ (influence of sectional area A_{cp}), ٤ (influence of the nominal stirrup strength $\rho_v.f_{yt}$), and ٥ (influence of the nominal longitudinal steel strength $\rho_\ell.f_{y\ell}$). For the ACI ٣١٨M-٠٥ Code[٢١] method, there is a significant rise in the factor of safety with rising value of $\frac{y}{x}$, while the safety factor of the proposed method – equation (١٥) is not influenced by variation of $\frac{y}{x}$ value- Fig.٢. The influence of A_{cp} is indicated in Fig.٣ which shows that for ACI ٣١٨M-٠٥[٢١] method, the factor of safety decreases with increasing A_{cp} value. In contrast, the safety factor of the proposed method is approximately constant with variation of A_{cp} value.

Figs. ٤ and ٥ show clear trends for the overestimation of the influence of the nominal steel strength ($\rho_v.f_{yt}$ and $\rho_\ell.f_{y\ell}$) by the ACI ٣١٨M-٠٥[٢١] method. On the other hand, the proposed method shows no variation in the safety factor with rising value of $\rho_v.f_{yt}$ (ranging between ١,٧-٧,٢٥) MPa, and $\rho_\ell.f_{y\ell}$ (ranging between ٢,٨٨-٢٤,٧) MPa.
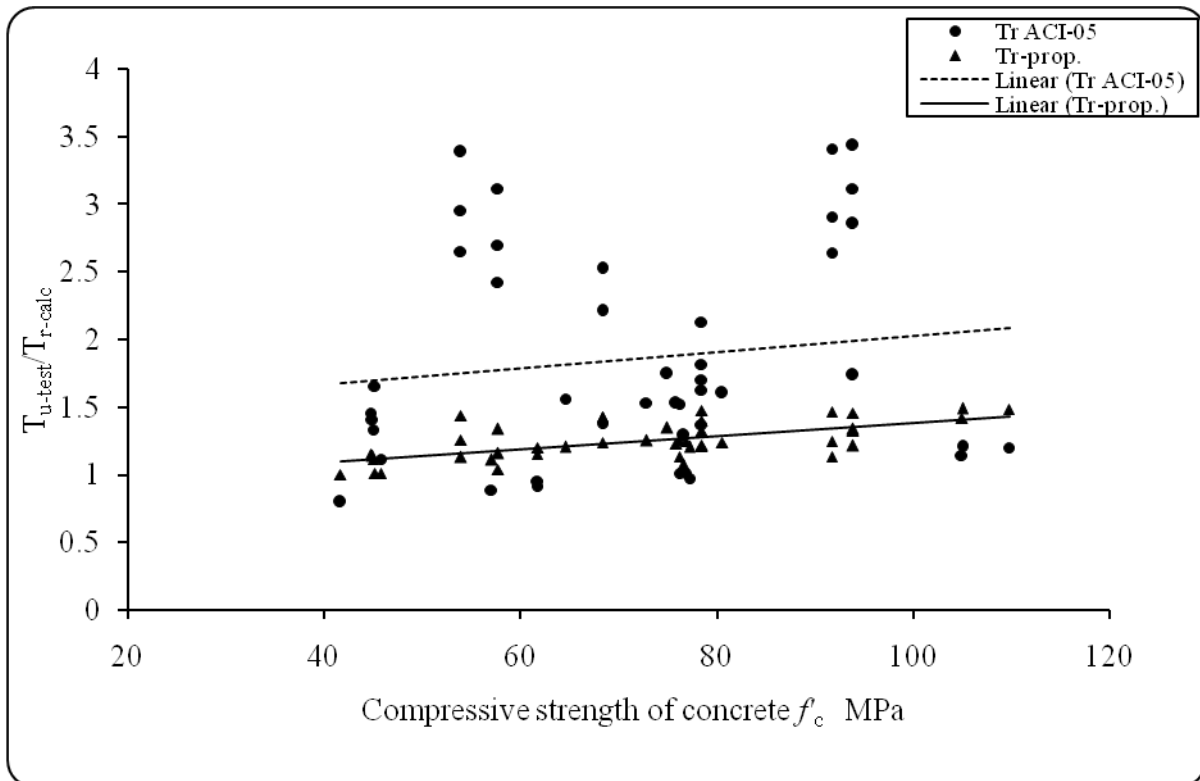

Fig.١ - Influence of compressive strength of concrete f'_c on test results
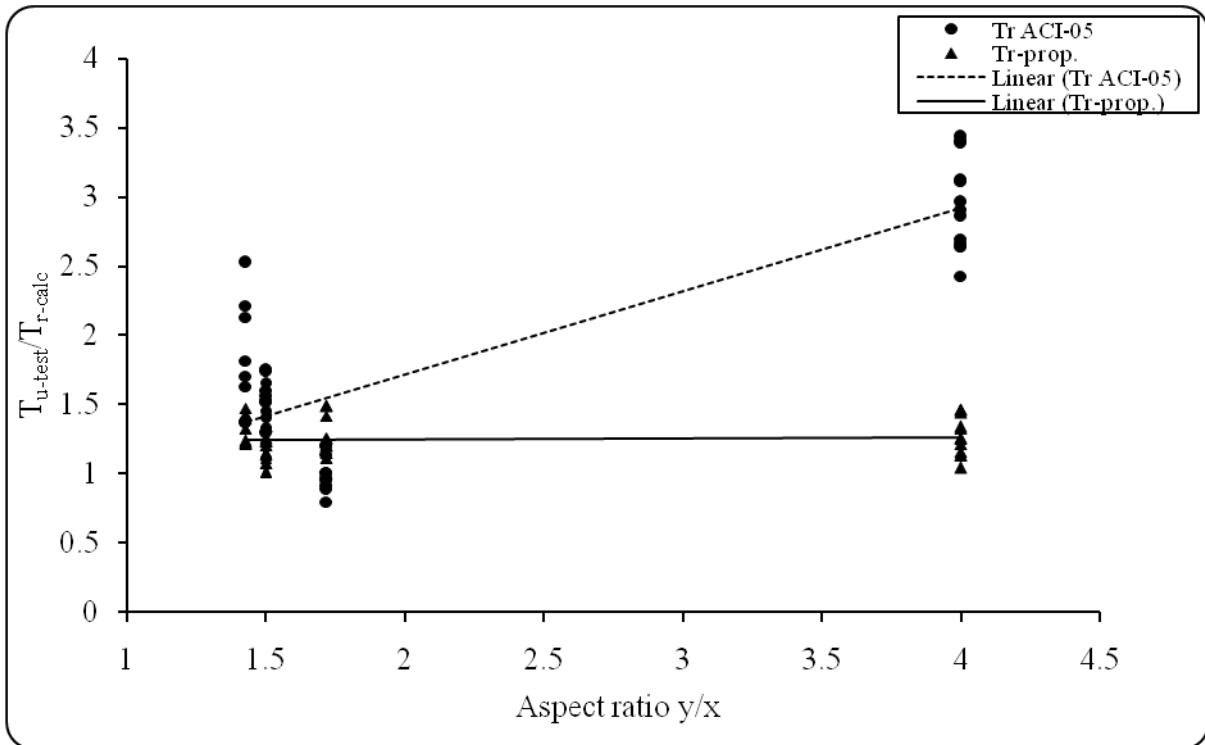
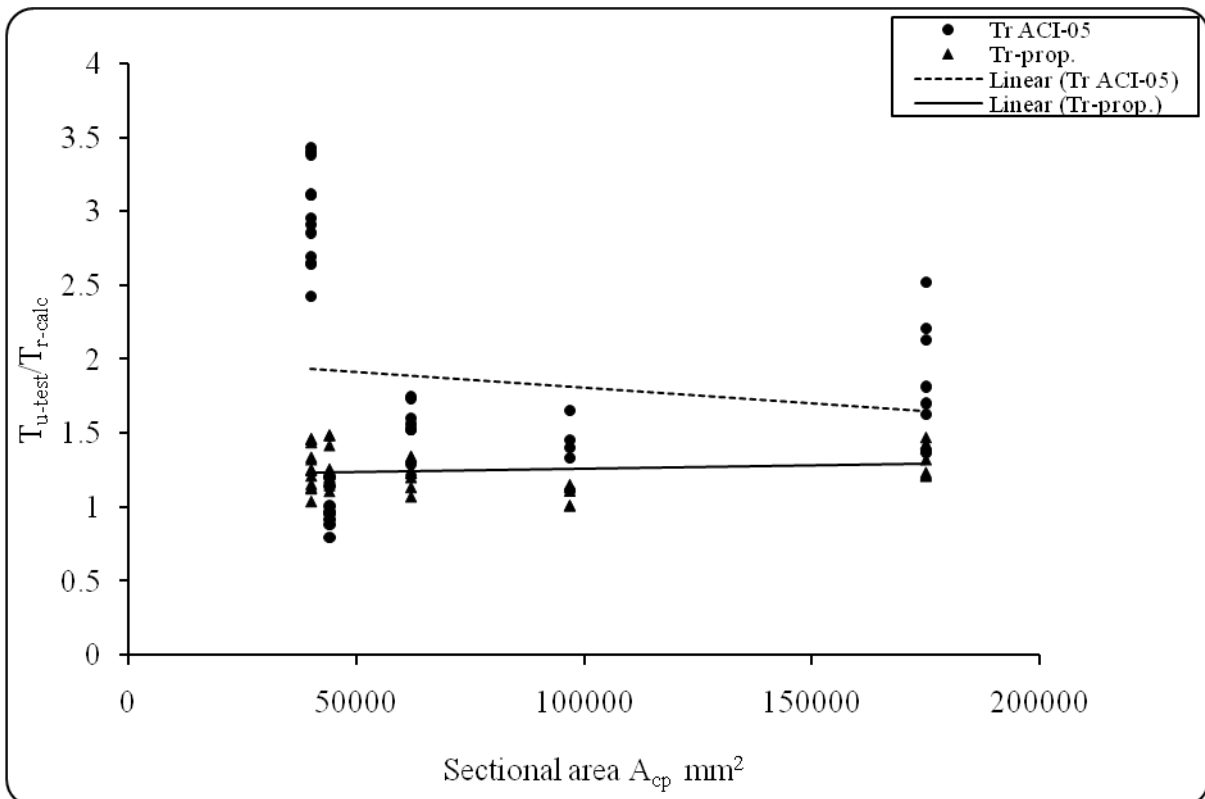**Fig.٢ - Influence of aspect ratio *y/x* on test results**

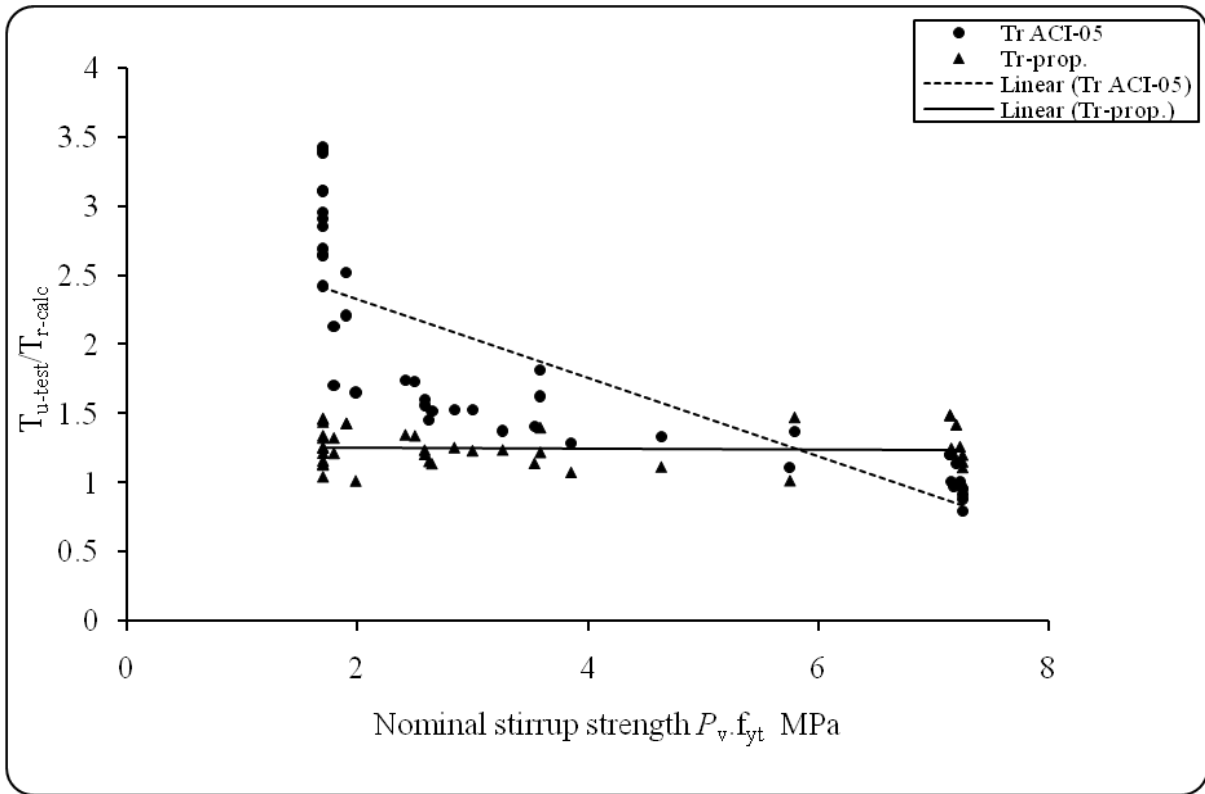**Fig.٣ - Influence of sectional area *Acp* on test results**

Fig.٤ - Influence of nominal stirrup strength $\rho_v.f_{yt}$ on test results
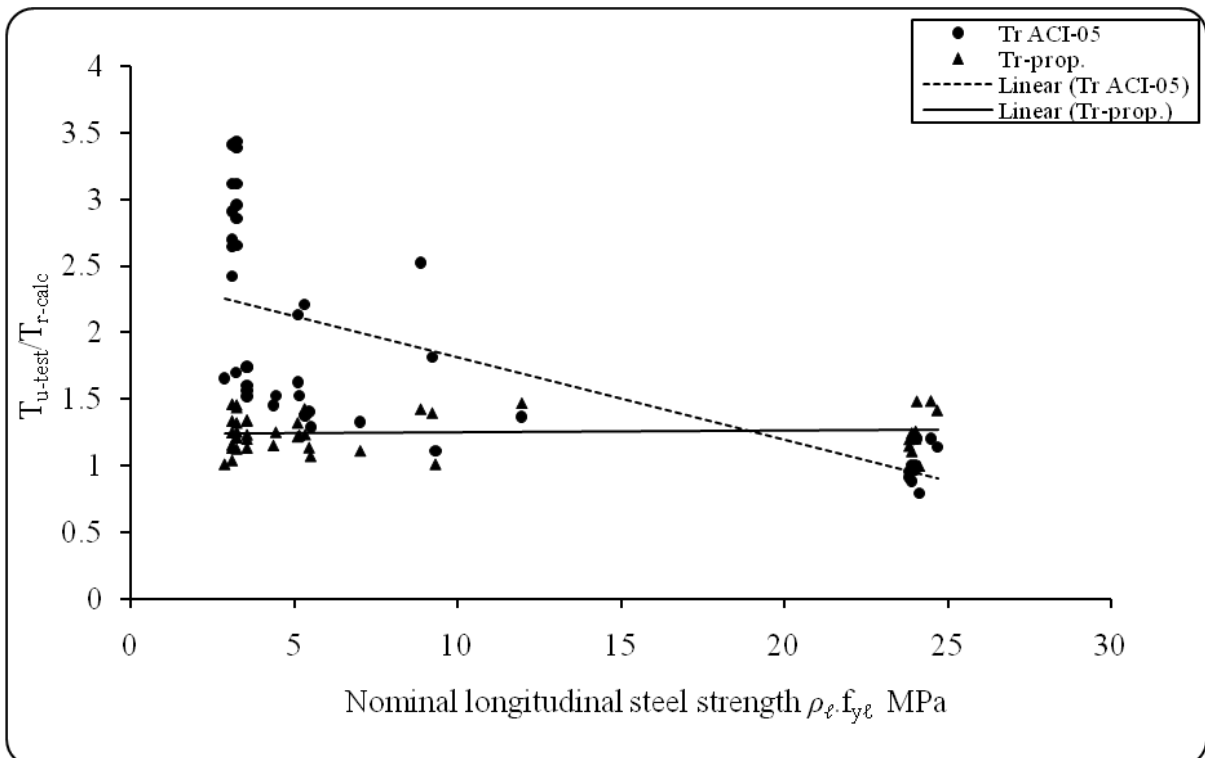

Fig.٥ - Influence of nominal longitudinal steel strength $\rho e.f_{y e}$ on test results

CONCLUSIONS:

Based on this work, the following conclusions are made:

١. A simple equation (١٤) is presented to estimate cracking torsional moment T_{cr} in HSC rectangular section beams.

٢. Another equation (١٥) is suggested for predicting torsional resistance moment T_r of such beams. This method agrees with the recent trend of space truss analogy of shear flow that bases strength only on the contribution of reinforcement- as in ACI ٣١٨M-٩٥ and later ACI Code versions, Canadian, AASHTO-LRFD, and EURO methods.

٣. The existing methods, give COV values between (٩,٣٨-٩,٤٨) percent for the ratio ($T_{cr\text{-}test}/T_{cr\text{-}calc}$), while the proposed equation (١٤) leads to a COV value of (٨,٩٣) percent for this ratio.

٤. The COV value of the existing code design methods ranges between (٢٤,١٩-٤٢,٨٢) percent for the ratio ($T_{u\text{-}test}/T_{r\text{-}calc}$). On the other hand, a significant reduction in COV value has been obtained when the proposed equation (١٥) was applied, which led to the best value of COV-١١,١٧ percent for this ratio.

٥. The proposed method – equation (١٥) is similar to the EURO one- equation (١٣), with one major difference. Proposed equation (١٥) uses powers of values less than ٠,٥ for ٥ parameters: A_t, A_ℓ, P_h, f_{yt}, and $f_{y\ell}$. Therefore, for the ratio ($T_{u\text{-}test}/T_{r\text{-}calc}$), equation (١٥) gives low value of essentially ٠,٩٩٩ ≅ ١ (Table ٣). In contrast, EURO method has respective values of ٠,٣٩٤ and ١٤.

٦. The latest code design method (ACI ٣١٨M-٠٥) has the highest mean value (at ١,٨٦) of all the other methods and the highest ratio of $T_{u\text{-}test}/T_{r\text{-}calc}$ (at ٣,٤٣). Despite this, ACI ٣١٨M-٠٥ Code [٢١] method leads to ٥ unsafe ratios, with a low value of (٠,٨).

٧. For a range of f'_c between (٤١,٧-١٠٩,٨) MPa, the proposed equation (١٥) gives safe prediction (Fig.١), as well as a rising factor of safety with increasing f'_c. This is considered useful, since the number of available HSC tests in torsion is limited, compared to normal strength concrete ($f'_c < $ ٤١,٣ MPa).

٨. Figs. ٢,٣,٤, and ٥ show that the factor of safety of the proposed equation (١٥) is not influenced by rising values of $\dfrac{y}{x}$, A_{cp}, $\rho_v.f_{yt}$, and $\rho_\ell.f_{y\ell}$. On the other hand, the safety factor of the ACI ٣١٨M-٠٥ Code method increases with rising value of   and decreases with rising values of A_{cp}, $\rho_v.f_{yt}$, and $\rho_\ell.f_{y\ell}$. This may be that the proposed equation (١٥) relates to these ٤ factors more closely than ACI ٣١٨M-٠٥ method with practical tests.

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