

Finite Element Method for Improving Soft Soil Underneath a Ballast Railway Track

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Abstract

It is always recommended to improve the properties of the soft soil beneath the railway networks often in such cases to increase its ability in bearing different applied loads and to control the expected generated settlements. The most methods used to improve the soil is by using a ballast layers with or without reinforced single geogrid layer or a geogrid layers at different spacing. This study presents a three-dimensional finite element analysis for soft soil underneath a ballast railway track by using a finite element program (ANSYS) which considers in these days the most software using in many engineering applications and most completeness. Twenty four models were created using a nonlinear three-dimensional finite element to study the effect of ballast thickness, mechanical properties of soft soil (undrained shear strength and modulus of elasticity), geogrid layer reinforcement to improve the soft soil. The ballast, soft soil and steel plates were modeled by using 8-nodes brick element with three degree of freedom per node. While, 4-nodes Shell element with six degree of freedom per node was used to represent the geogrid layer under and between ballast. The results show that increasing the undrained shear strength (C_u) and modulus of elasticity (E) lead to decreasing the settlement of soft soil and increasing the ultimate load. Increasing ballast thickness lead to decreasing the settlement of soft soil and increasing the ultimate load, this means that modulus of elasticity and shear strength playing main role to controlling in settlement of soft soil (ultimate displacement under plate loading) and ultimate load

Key words: *Finite Element, Ballast, Railway, ANSYS, Geogrid, Reinforced soft soil, Improvement*

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الخلاصة

يوصى بتحسين خواص التربة تحت شبكة خطوط سكك الحديد لزيادة قابلية تحملها لمقاومة الاحمال المختلفة وللسيطرة على الهبوط المتولد. ان اكثر الطرق المستخدمة لتحسين التربة هي باستخدام طبقات من حجر التحكيم المسلحة مع اوبدون طبقة منفردة من المشبكات اللدانية او المسلحة بطبقات من المشبكات اللدانية وبمسافات مختلفة. تتناول هذه الدراسة بحثا نظريا للتحليل بطريقة العناصر المحددة للتربة الضعيفة المثبتة اسفل حجر التحكيم لخط سكة حديد باستخدام برنامج العناصر المحددة (ANSYS) الذي يعتبر في الوقت الحاضر احد البرامج الواسعة الاستعمال في العديد من التطبيقات الهندسية واكثرها شمولاً. تم في هذه الدراسة بناء اربعة وعشرون نموذجا رياضيا باستعمال طريقة العناصر المحددة ثلاثية الأبعاد اللاخطية لتقصي تاثير سمك حجر التحكيم (Ballast)، خواص التربة (معامل مقاومة القص (C_u) ومعامل المرونة (E))، التقوية باستخدام المشبكات اللدانية (Geogrid) على تحسين و تثبيت التربة بالاستفادة من برنامج العناصر المحددة ANSYS. تم استخدام عنصر صلب ثلاثي الأبعاد ذو ثمانية عقد مع ثلاثة درجات للحرية في كل عقده لتمثيل حجر التحكيم (Ballast)، بينما تم استخدام عنصر صلب ثلاثي الأبعاد ذو ثمانية عقد مع ثلاثة درجات للحرية في كل عقده لتمثيل طبقات التربة والصفائح الحديدية. بينما تم استخدام عنصر قشري ثلاثي الأبعاد ذو اربعة عقد مع ستة درجات للحرية في كل عقده لتمثيل المشبكات اللدانية اسفل وداخل طبقات حجر التحكيم. اظهرت النتائج ان الزيادة في قيم معامل القص (C_u) ومعامل المرونة (E) قلل من هبوط التربة وكذلك ادى الى زيادة التحمل الاقصى، كذلك بينت النتائج ان زيادة سمك طبقة حجر التحكيم قلل من هبوط التربة وكذلك ادى الى زيادة التحمل الاقصى، هذا يعني ان هذه المعاملات تلعب دورا أساسيا في السيطرة على هبوط التربة (الإزاحات القصوى المتولدة أسفل صفيحة التحميل) وكذلك الاحمال القصوى.

١- Introduction

Soft clays are recent alluvial deposits probably formed within the last ١٠,٠٠٠ years characterized by their flat and featureless ground surface. (Brand and Bernner, ١٩٨١) are identified by their low undrained shear strength ($C_u < ٤٠$ kPa) and high compressibility (C_c between ٠,١٩ to ٠,٤٤). They are found at high natural moisture content, typically ranging from (٤٠-٦٠%) with plasticity index ranging from ٤٥-٦٥%. (Broms, ١٩٨٧). Soils with such characteristics create serious problems to geotechnical engineering associated with stability and settlements problems. Many techniques are available to improve such soils based on

reducing the water content by several mechanisms such as sand drains, wicks, electrical osmosis, geogrid and thermal treatments. On the other hand some other techniques are also developed towards improving the engineering properties of these clays by introducing sand compaction piles or stone columns, where holes with specific depth and diameter are made within the soil in a grid form and backfilled with granular material. The ANSYS computer program is a large-scale multipurpose finite element program which may be used for solving several classes of engineering analyses. The analysis capabilities of ANSYS include the ability to solve static and dynamic structural analyses. The program contains many special features which allow nonlinearities or secondary effects to be included in the solution, such as plasticity, large strain, hyperelasticity, creep, swelling, large deflections, contact, stress stiffening, temperature dependency, material anisotropy and radiation.

2-Aims of the Study

The main aims of this study are to investigate theoretically the improvement of soft soil reinforced with geogrid layers with or without ballast. The effect of soft soil characteristics (angle of friction and cohesion), thickness of ballast layers and presence or absence of geogrid on ultimate load capacity, vertical displacement (settlement) and mode of failure under monotonic loads (pressure) will be investigated. The work includes the following two main categories:

- 1- To implement a nonlinear finite element procedure to analyze all adopted models.
- 2- To assess the finite element analysis results.

3-Material Properties Modeling

3-1-Soft Soil

In general, there is no clear definition of soft clay. There are several approaches which can be used in identifying and classifying of soft soil. Geotechnical design and execution of civil engineering structures on soft to very soft soil are usually associated with substantial difficulties since this type of soil is sensitive to deformation and possesses very small shear strength (Kempfert and Gebreselassi, 2006).

3-2-Shear Strength Parameters of soil (C_u and ϕ)

Shear strength parameters of soil (C_u and ϕ) can be determined experimentally by Triaxial testing in clay (consolidated undrained test-CU). The values of friction angle (ϕ) and Cohesion (C_u) are obtained by drawing a common tangent to effective-stress Mohr's circles (Mohr-Coulomb envelope) for various tests.

3-3-Mechanical Properties of Ballast

3-3-1-Compression Strength of Ballast

The compressive test strength of Ballast should be performed on cubic samples measuring (7 cm) on each edge. For each test, four samples shall be taken from quarry face, in such way

as to reflect parent rock characteristics. The average compression strength of four samples shall not be less than 100 Kg/cm² (10MPa).

3-3-2-Tensile Strength of Ballast

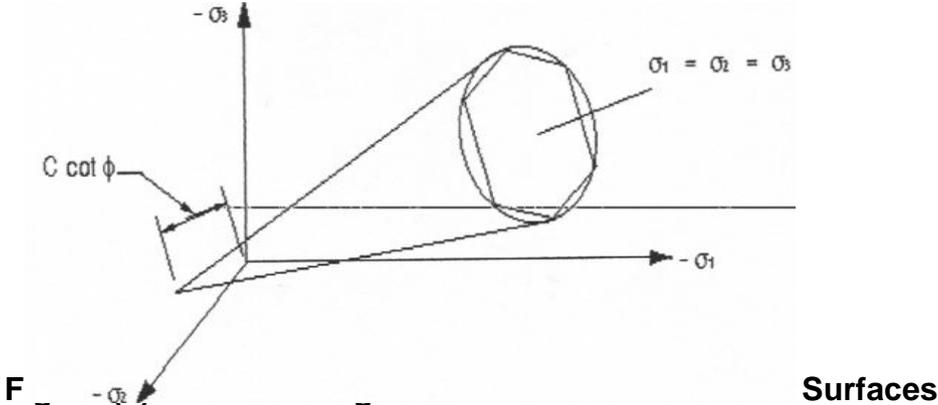
Experimental results (McDowell and Bolton) show that the mean tensile strength (σ_f) of single particle can be considered as a function of average particle size (d) as shown in the following empirical equation:-

$$\sigma_f = \frac{F}{d^2} \dots\dots\dots (1)$$

Where (σ_f) is the characteristic tensile stress induced within particle at failure, (F) is the force applied and (d) is the particle size. It may be noted that the tensile strength of Ballast are ignored and not considered in the present study.

4-Failure Criteria for Soft Soil

(Drucker-Prager, 1959) yield criterion is widely used for finite element analysis of granular material problems (such as soil, gravel, sand, rocks....etc). In ANSYS program, the option uses the Drucker-Prager yield criterion is available with either an associated or non-associated flow rule. The yield surface does not change with progressive yielding, hence there is no hardening rule and the material is elastic-plastic. Figure (1) show comparison between Drucker- Prager and Mohr-Coulomb yield surfaces.



5-Failure Criteria for Ballast

The actual behavior and strength of ballast materials are very complex because they depend on many factors such as the physical and mechanical properties of the particles such as ballast

size, air voids, friction between particle and the nature of loading. No single mathematical model can describe the strength of real ballast materials completely under all conditions; so, simple models or criteria are used to represent the properties that are essential to the problem being considered.

(Willam and Warnke, 1975) developed a mathematical model capable of predicting failure for the solid cracking in tension and crushing in compression. In concrete applications, for example, the solid capability of the element may be used to model the concrete. Other cases for which the model is also applicable would be reinforced composites (such as fiberglass), and geological materials (such as rocks) (ANSYS, 2007). Figure (2) show the hydrostatic and deviatoric sections of Willam-Warnke model.

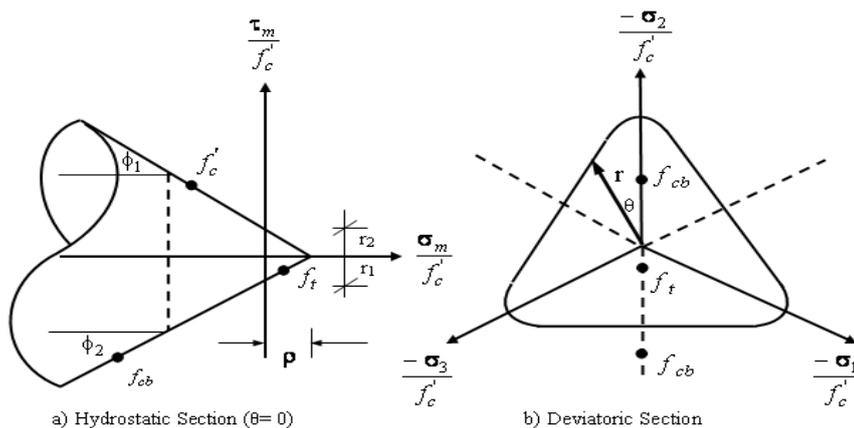


Figure (2) Failure Surface (Chen, 1982)

2-Failure Criteria for Geogrid and Steel Plate

For most metals, Von-Mises yield criterion is used because is simpler to use in theoretical application (Chen, 1982). This criterion assumes that failure (yielding) occurs when the octahedral shear stress (τ_{oct}) reached its critical value. Mathematically, this criterion can be expressed in the following form:-

$$f(J_v) = J_v - k^2 = 0 \quad \dots \dots \dots (2)$$

Where:-

$$k = \text{Failure (yield) stress in pure shear} = \frac{1}{\sqrt{3}} f_y$$

Figure (3) shows the Deviatoric and Meridian sections corresponding to Von-Mises failure surface.

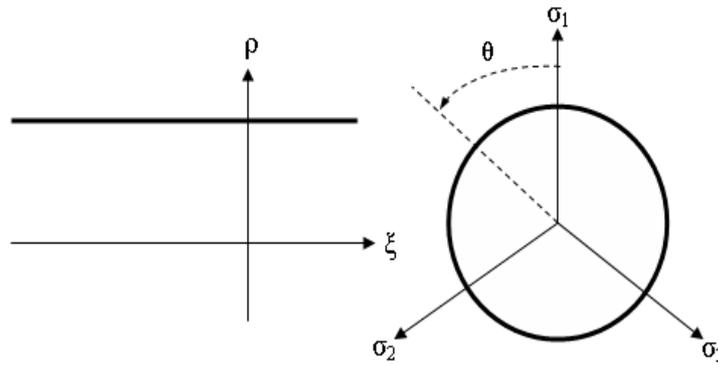


Figure (r) Meridian and Deviatoric Sections for Von-Mises Criterion

3D-Finite Element Modeling

As mentioned before, the ANSYS computer program was utilized for analyzing all models. Model components encountered throughout the current study, corresponding finite element representation and corresponding elements designation in ANSYS are presented in Table (1)

Table (1) Finite Element Representation of Model Components

Model Component	Finite Element Representation	Element Designation in ANSYS
Ballast (Rocks)	8-Nodes Brick Element (3-Translation DOF per node)	SOLID-20
Soft Soil	8-Nodes Brick Element (3-Translation DOF per node)	SOLID-20
Steel Plates		
Geogrid	8-Nodes Shell Element (3-Translation & 3-Rotational DOF per node)	SHELL-281

Three dimensional solid elements (SOLID-20 in ANSYS) are used for three dimensional modeling of solid structures such as reinforced concrete and geological materials (such as rocks and soil). The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y and z directions. The element has plasticity, creep, swelling, stress stiffening, large deflection and large strain capabilities. It may be noted that, in the present study, this element is used to model soft soil layers, Figure (xi).

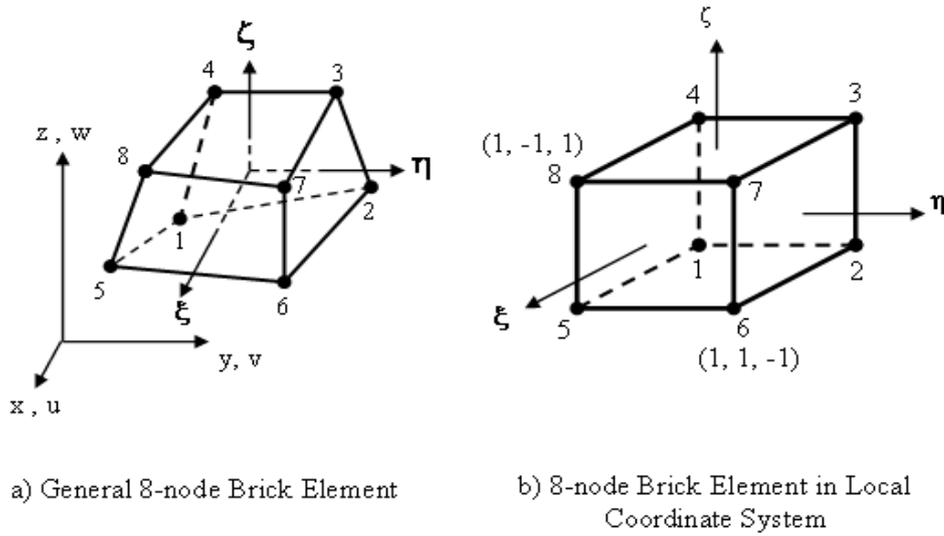


Figure (4) Three Dimensional Solid Elements (Solid-20 in ANSYS)

Four nodes shell element (SHELL-20) in ANSYS) is used for analyzing thin to moderately-thick shell structures. It is a 20-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes as shown in Figure (5).

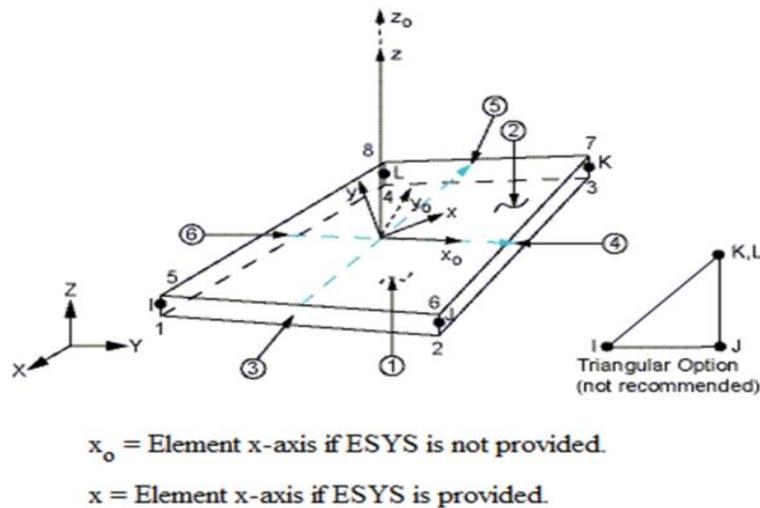


Figure (5) Four Nodes Shell Element (Shell-20 in ANSYS)

(If the membrane option is used, the element has translational degrees of freedom only). This element is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses. SHELL-181 may be used for layered applications for modeling laminated composite shells or sandwich construction. In the present study, this element is used to model Geogrid layers.

Three dimensional reinforced concrete solid (SOLID-20 in ANSYS), is used for the three dimensional modeling of solids with or without reinforcing bars. The solid is capable of cracking in tension and crushing in compression. In structural applications, for example, the solid capability of the element may be used to model the concrete. Other cases for which the element is also applicable would be reinforced composites (such as fiberglass), and geological materials (such as rock). The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions as shown in Figure (1). It may be noted that, in present study, this element were used to model all ballast layers.

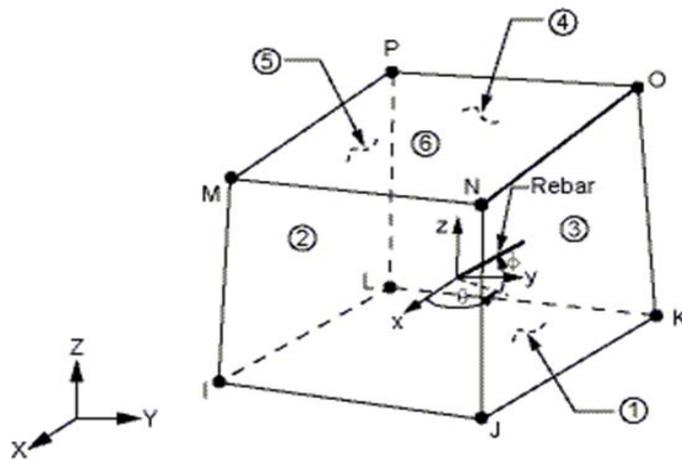


Figure (1) Three Dimensional Reinforced Concrete Solid (Solid-20 in ANSYS)

4-Numerical Applications

4-1 Geometry and Model Creation

In actual field condition, the soil is usually of infinite extent both in horizontal and vertical directions. In the finite element idealization the horizontal boundary of the soil blocks in the (x) and a (y) direction is required. The dimensions of the soft soil considered in the analysis were (1000x1000x1000 mm) for the length (in x-direction), width (in z-direction) and depth (in y-direction) respectively, Figure (7).

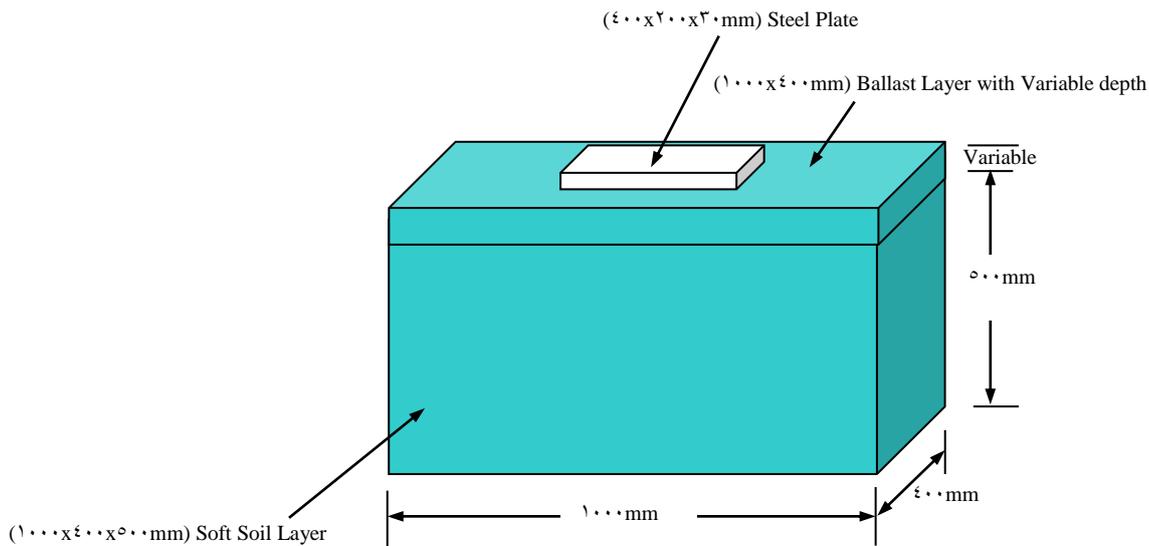


Figure (7) Dimensions of Adopted Models

All dimensions of soft soil layer have been kept constant for all analyses. While, the depth (thickness) of ballast layers were variable and depend on considered case (state). It may be noted that, the adopted dimensions were employed in the experimental work done by (Abbawi, 2010).

4-2 Loading and Boundary Conditions

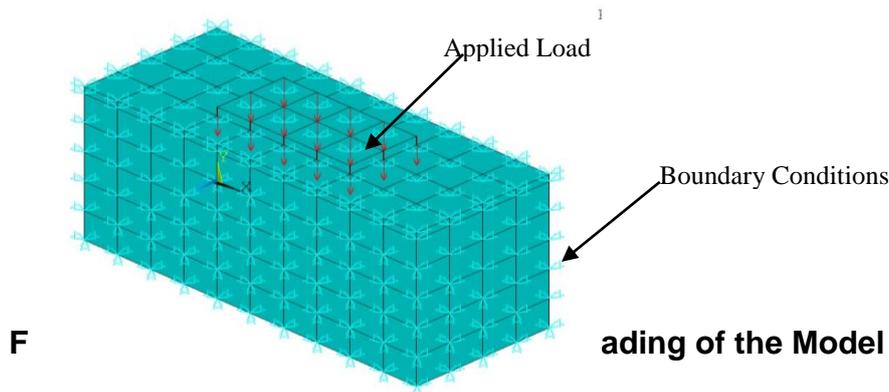
Displacement boundary conditions (which represent the conditions at the interface of model) are needed to constrain the model to get a unique solution. To ensure that the model acts the same way as a real case, boundary conditions need to be applied at all sides of the model, and where the loadings exist. The word load in ANSYS includes boundary condition and external or internal applied force (different types of load available in ANSYS such as structural, thermal, fluid...). The type of loading were used in this study was concentrated loads with different values; Due to load concentration on ballast elements, crushing of the ballast started to develop in the elements located directly under the loads. Subsequently, adjacent ballast elements crushed within several load steps. As a result, the model showed a large

displacement, solution diverged and finally, the finite element model fails prematurely. Therefore, to prevent this phenomenon, two techniques were used:-

- 1) -Finer mesh was used under applied load.
- 2) -Steel plates were used under load.

In the present study, the second technique was adopted, and the employed boundary conditions were as follows:-

- 1. Hinges, at the side of model in x and z-directions and, rollers in y-direction, Figure (A).
- 2. Fixed at the bottom face of model (restrained the nodes in x, y and z-directions).



1-3-Finite Element Modeling

A twenty four model, divided into four groups were created in the present study as shown in Table (). It may be noted that each model was designated in a way to refer to Soft soil layer, first Ballast layer, Giogrid layer, second Ballast layer, undrained shear strength ($C_u=9$ kPa and $C_u=20$ kPa) and thickness of ballast layer (20, 50, 70 and 100 mm). Therefore, the model (SBGB-1), for example, is a finite element model consists of soft soil layer, first ballast layer, giogrid layer located at (20 mm) from the top layer of ballast, second ballast layer, undrained shear strength of ($C_u=9$ kPa) and thickness of ballast layer (50 mm).

1-4 Models Parameters

The finite element models adopted in this study have a number of parameters, which can be classified into four categories:

- i- Soft soil property parameters, Table 1
- ii- Ballast property Parameters, Tables 2
- iii- Geogrid property parameters, Table 3
- iv- Steel plates property, Table 4

Table (٢) Soft Soil Property Parameters

Parameter	Definition	value	Note
Cu	Unrained shear strength (kPa)	٩,٠	Assumed
		٢٥	
E	Elastic Modulus of Elasticity (MPa)	٤,٥	$E=٢٥ \cdot C_u - ٥ \cdot C_u^*$
		١٢,٥	
v	Poisson's ratio	٠,١٥	*
ϕ	Angle of Friction	٠	

* Das, (٢٠٠٦)

Table (٣) Ballast Property Parameters

Parameter	Definition	value	Note
f'_c	Ultimate Compressive Strength (MPa)	٤٨	Iraq Railway Company
E	Elastic Modulus of Elasticity (MPa)	١٣٠	
v	Poisson's ratio	٠,٤٥	
β_c	Shear transfer Coefficient	٠,٢٢	Assumed
β_o	Shear transfer Coefficient	٠,٢	

Table (٤) Steel Plate Property Parameters*

Parameter	Definition	value	Note
f_y	Ultimate tensile strength (MPa)	٤٢٠	Assumed
E	Elastic Modulus of Elasticity (MPa)	٢٠٠×١٠^٣	
v	Poisson's ratio	٠,٣	
t	Thickness (mm)	٣٠	

*Saudi Arabian stander organization (SASO) test method ISO ١٠٣١٩

Table (9) Geogrid Property Parameters

Parameter	Definition	value	Note
f_t	Peak tensile strength (N/mm)	13,0	
E	Elastic Modulus of Elasticity (MPa)	20	
ν	Poisson's ratio	0,3	
t	Thickness (mm)	3	Assumed

9-Results and Discussion

After creating the model and entering all associated model parameters, the analysis is performed. The ANSYS divides the load into a number of sub-steps and performs the iteration for each sub-step until reaching the convergence. Figures (9) and (10) show the deformed shape of model for two undrained shear strength when the undrained shear strengths of untreated soil changed from (9kPa) to (20kPa), the modulus of elasticity increased and the load capacity increased for about (16%), while, the settlement decreased for about (47%). This means the undrained shear strengths represent important parameters to improve soil and as a result, the load capacity increased. Table (6) shows the result and Figure (11) shows the effect of undrained shear strength and modulus of elasticity on the load-settlement relationship.

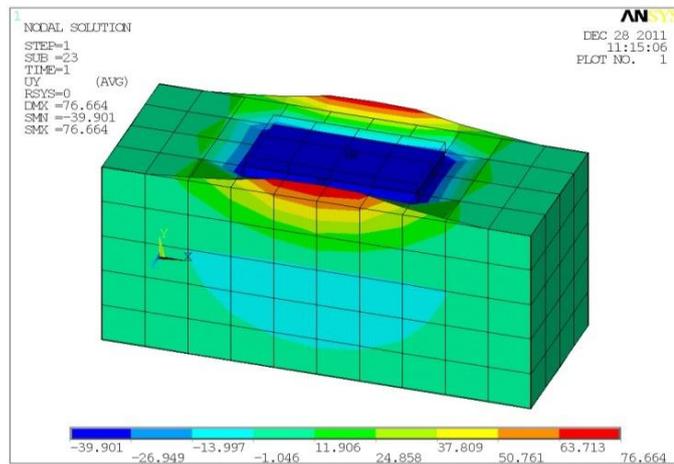


Figure (9) Failure Mode of Untreated Soil Model S-1

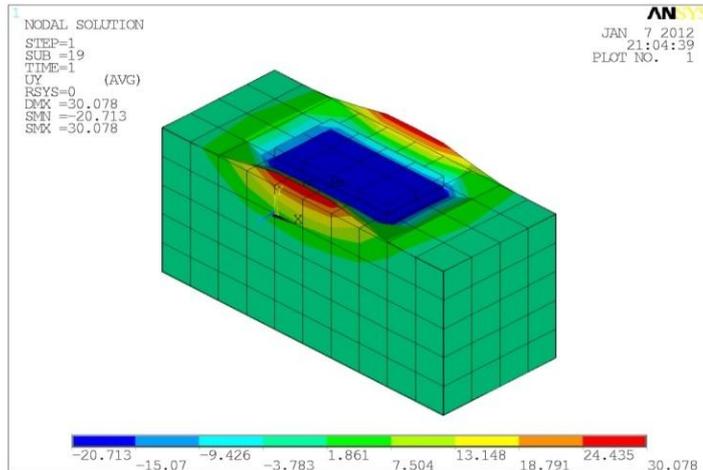


Figure (10) Failure Mode of Untreated Soil Model S-2

Table (1) Ultimate Load and Maximum Settlement for Group-1

Group	Model	E (kPa)*	P _u (kN)	(P _u) _i / (P _u) _R	S (mm)	(S) _i / (S) _R
G-1	S-1	2100	8.0	-	40	-
	S-2	40000	20.8	2.6	21	0.53

*From Equation $E=200 \cdot C-0.00 \cdot C$ (Das, 2006)

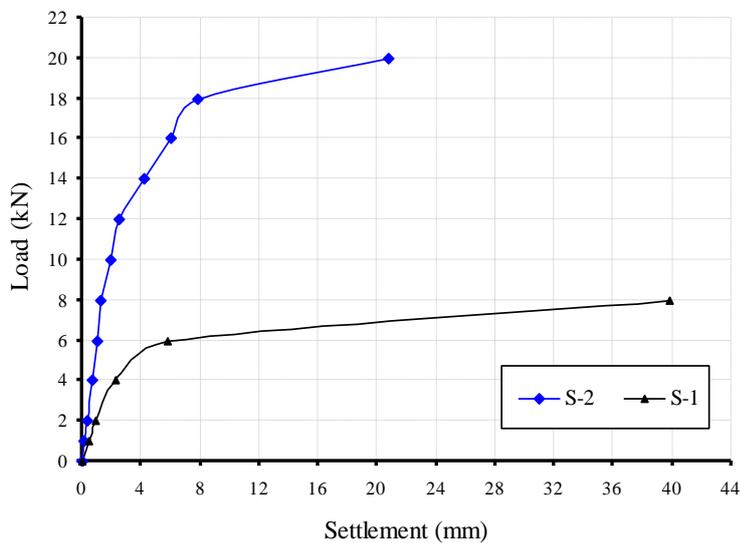


Figure (11) Load-Settlement Curve for Group-1

The second group consist of eight models (SB-1, SB-2, SB-3, SB-4, SB-5, SB-6, SB-7, and SB-8) performed with ballast layer overlaying the soft soil. The eight modes were performed using different ballast thickness (H) of (20, 50, 70 and 100 mm). Four models were performed on each of the two undrained shear strengths (9kPa) and (20kPa). Table (2) shows

comparison between the ultimate loads from the finite element analysis. Figures (12) and (13) shows the relationship between the applied load (P) and the corresponding settlement (S) for the models of the second group (SB-1, SB-2, SB-3, and SB-4) and (SB-5, SB-6, SB-7, and SB-8) were constructed and compared with reference models (S-1 and S-2)

Table (v) Ultimate Load and Maximum Settlement for Group-2

Group	Model	(P _u) _R (kN)	P _u (kN)	(P _u) _i /(P _u) _R	(S) _R (mm)	S (mm)	(S) _i /(S) _R
G-2	SB-1	8.0	23	2.88	4.0	16.33	0.41
	SB-2		30	3.75		17.19	0.43
	SB-3		43	5.38		24.47	0.61
	SB-4		61	7.63		34.43	0.86
	SB-5	20.8	30	1.44	21	8.87	0.42
	SB-6		41	1.97		8.53	0.41
	SB-7		52	2.50		10.41	0.50
	SB-8		66	3.18		12.94	0.62

*(P_u)_R= Ultimate Load of Untreated Soil for Two Undrained Shear Strength (S-1 & S-2)

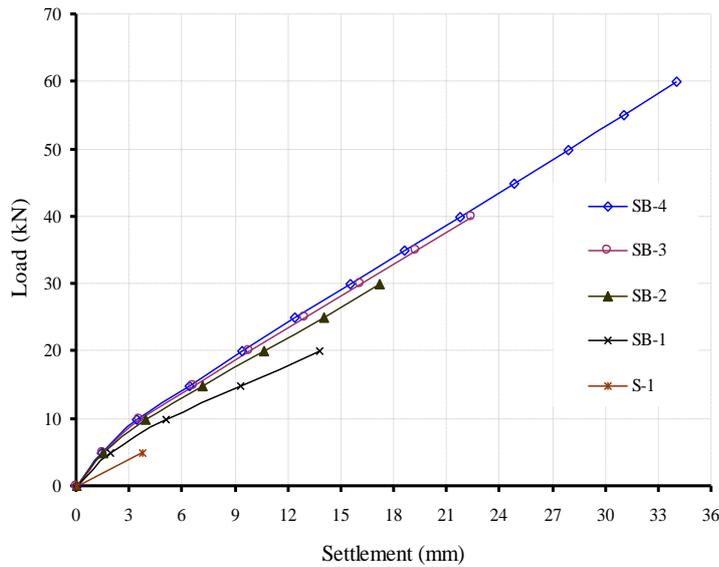


Figure (12) Load-Settlement Curves for Group-2 and Untreated Model (S-1)

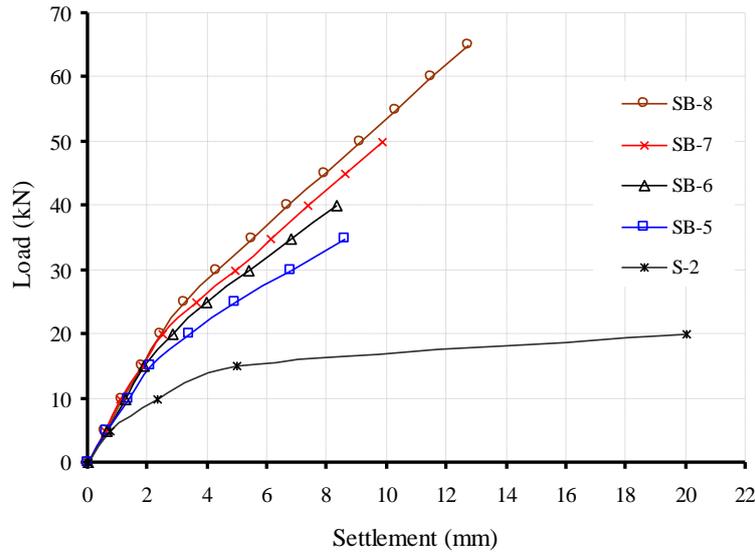


Figure (13) Load-Settlement Curves for Group-2 and Untreated Model (S-2)

The third group consist of eight models were performed with ballast layer reinforced with geogrid overlying the soft soil. These models were performed using different ballast thickness (H) of (20, 30, 40 and 50 mm). Four models were performed on each of the two undrained shear strengths (1kPa) and (2kPa).

Initially, a single layer of geogrid was placed along the interface plane between the ballast and soft soil. Figures (14) and (15) show the models reinforced with (20mm) ballast and a geogrid layer located between the soft soil and ballast layer and the effect of geogrid in settlement and ultimate load capacity for two undrained shear strength. Table (6) shows comparison between the ultimate loads from the finite element analysis

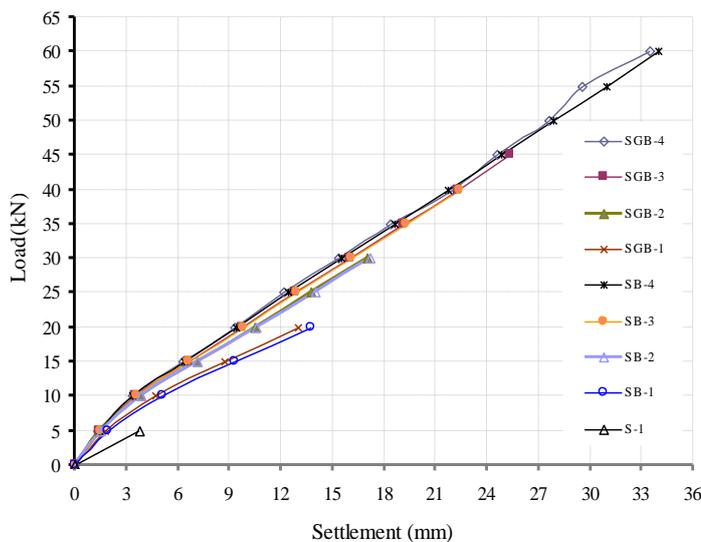


Figure (14) Load-Settlement Curves for Groups-2&3 and untreated model (S-1)

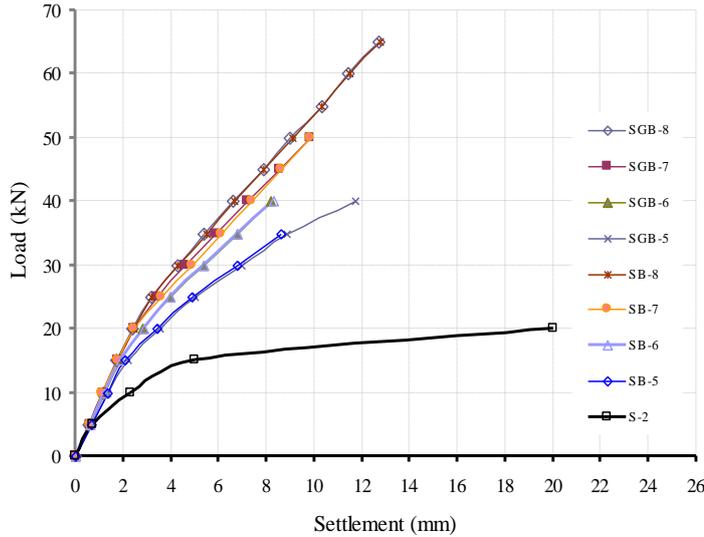


Figure (10) Load-Settlement Curves for Groups- 2 & 3 and Untreated model (S-2)

Table (8) Ultimate Load and Maximum Settlement for Group-3

Group	Model	$(P_u)_R$ (kN)	P_u (kN)	$(P_u)_i / (P_u)_R$	$(S)_R$ (mm)	S (mm)	$(S)_i / (S)_R$
G-3	SGB-1	8,0	20	3,13	40	16,0	0,41
	SGB-2		32	4,00		18,6	0,47
	SGB-3		40	5,64		20,7	0,64
	SGB-4		63	7,88		36	0,9
	SGB-5	20,8	43	2,07	21	13,2	0,63
	SGB-6		43	2,07		9,0	0,40
	SGB-7		50	2,64		11	0,44
	SGB-8		68	3,27		13,0	0,64

*($P_u)_R$ = Ultimate Load of Untreated Soil for Two Undrained Shear Strength (S-1 & S-2)

The fourth group consist of six models were performed with ballast layer reinforced with geogrid layer in the top and these models were performed using ballast thickness (H) of (20, 30 and 40 mm). The models performed by placing the geogrid layer at a distance (20mm) below the level of ballast thickness. Figures (16) and (17) shows the results demonstrate a substantial increase the ultimate load with increasing thickness of ballast due to the distribution of the applied load. Table (9) shows comparison between the ultimate loads from the finite element analysis.

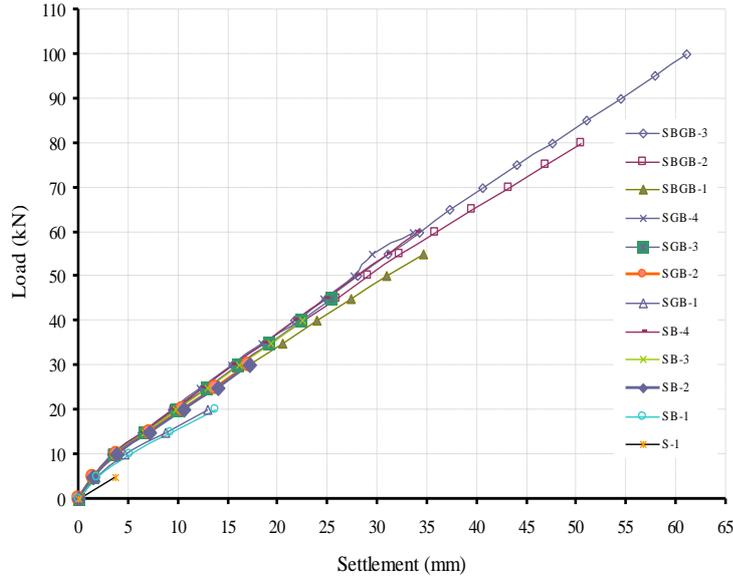


Figure (16) Load-Settlement Curves for Groups-2 & 3 & 4 and Untreated Model (S-1)

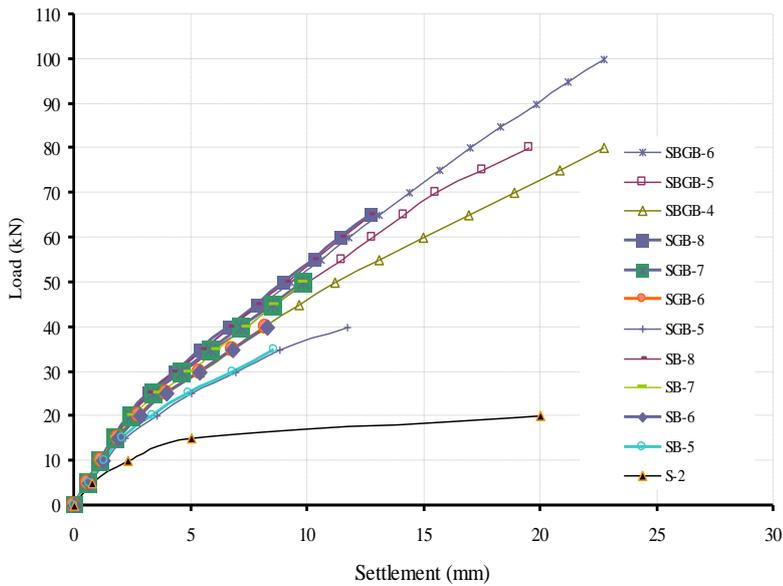


Figure (17) Load-Settlement Curves for Groups-2 & 3 & 4 and Untreated Model (S-2)

Table (1) Ultimate Load and Maximum Settlement for Group-ε

Group	Model	(P _u) _R (kN)	P _u (kN)	(P _u) _i /(P _u) _R	(S) _R (mm)	S (mm)	(S) _i /(S) _R
G-ε	SBGB-1	8.0	56	7.00	ε.	30	0.88
	SBGB-2		82	10.30		52	1.30
	SBGB-3		110	13.80		62	1.00
	SBGB-ε	20.8	83	3.99	21	26	1.24
	SBGB-ο		84	4.04		21	1.00
	SBGB-6		110	5.29		28	1.33

*(P_u)_R= Ultimate Load of Untreated Soil for Two Undrained Shear Strength (S-1 & S-2)

10- Conclusions

Based on the results obtained from the finite element analysis for improvement of soft soil reinforced with or without giogrid, the following conclusions are presented:-

1-The vertical displacement (settlement) under the applied load decreases with the increase of shear strengths (C_u). Increasing of soil shear strength improve the load carrying capacity significantly. This enhancement starts even from the lower load and increases with increase in load.

2-The vertical displacement (settlement) under the applied load decreases with the increase of modulus of elasticity (E) of the soil. Increasing of soil modulus improve the load carrying capacity significantly.

3-The maximum vertical displacement under the applied load decreases with the increasing of the ballast thickness.

ε- Presence of giogrid layers leads to reduce the vertical displacement (settlement), while the corresponding load carrying capacity increased significantly. The uniformly oriented giogrid and its ability to improve soft soils cause an increase in the load carrying capacity. This was combined with the ability of ballast layer to sustain larger compressive force at advanced stages of loading.

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