

Numerical Study Of Natural Convection In Annular Enclosure With Linearly Varying Wall Temperature

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Abstract:

Natural convection heat transfer with two-dimensional in an annular enclosure is examined numerically for Rayleigh numbers rating from 103 to 105 and for aspect ratios 1,2,3.

The outer wall is heated using a linear temperature while the inner wall of the annular enclosure was cold, the top and bottom ends was adiabatic. The stream function–formulation was used in the mathematical model of the case under discussion where the differential equations were converted to algebraic equations using finite differences method. The numerical solution ability to calculate stream function, temperature and Nusselt number during the computational domain.

The numerical results showed that the main process of heat transfer is conduction for Rayleigh number less than $Ra \leq 103$, and convection for Rayleigh number greater than $Ra > 103$. Average Nusselt number is increased with aspect ratio $(L/(R_o - R_i))$ increasing for Ra ranging with (103-105) because increasing of heat transfer area. To ensure the reliability of the numerical solution a comparison of the current results with that of previous research, shows a good agreement between the results and this is confirms the validity and reliability of the current model.

Key words: natural convection, annular, enclosure, numerical, laminar.

دراسة عددية للحمل الحر في حيز حلقي مغلق مع تغير خطي لدرجة حرارة الجدار

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الجامعة التكنولوجية

الخلاصة :

تم إجراء دراسة عددية لانتقال الحرارة بالحمل الحر خلال حيز حلقي مغلق ثنائي البعد لأرقام رايلي 103 إلى 105 وينسب باعية 1,2,3 . الجدار الخارجي تم تسخينه باستخدام درجة حرارة خطية بينما الجدار الداخلي للحيز الحلقي كان بارداً والنهيات العليا والسفلى معزولة حرارياً .

تم استخدام صيغة دالة الانسياب والدوامية في النمذجة الرياضية للمسألة قيد البحث حيث تم تحويل المعادلات التفاضلية الى معادلات جبرية باستخدام طريقة الفروقات المحددة المركزية للحل العددي القابلية على حساب دالة الانسياب ودرجة الحرارة ورقم نسلت خلال المجال الحسابي .
 أظهرت النتائج العددية أن العملية الرئيسية لانتقال الحرارة هي بالتوصيل لرقم رالي أقل من أو يساوي 103 ، وانتقال الحرارة بالحمل لأرقام رالي اكبر من 103. رقم نسلت يتزايد مع زيادة نسبة الارتفاع إلى القطر $(L/(R_o - R_i))$ ضمن حدود رقم رالي (105-103) بسبب زيادة مساحة انتقال الحرارة .
 تم مقارنة النتائج الحالية مع نتائج البحوث السابقة حيث كان التوافق بين النتائج جيد .

Nomenclature

<u>Symbols</u>	<u>Meaning</u>	
AR	Aspect ratio $(L/(R_o - R_i))$	
C_p	Specific heat at constant pressure.	kJ/kg.k
g	Gravitational acceleration.	m/s^2
h	Heat transfer coefficient.	$w/m^2.k$
k	Thermal conductivity.	$w/m.k$
L	Length of annular enclosure	m
Nu	Nusselt number, (hL/k) .	
\overline{Nu}	Average Nusselt number	
p	Pressure.	N/m^2
Pr	Prandtl number, $(\mu C_p/k)$.	
R_i	Inner annuli radius	m
R_o	Outer annuli radius	m
Ra	Rayleigh number, $(Ra = \frac{\beta (T_h - T_c) L^3}{\alpha \nu})$	
T	Temperature.	K
U	Dimensionless Velocity component in r-direction.	
W	Dimensionless Velocity component in z-direction.	
R,Z	Dimensionless Cylindrical coordinates.	

Greek Symbol

α	Thermal diffusivity.	
β	Coefficient of thermal expansion.	
ϕ	General dependent variable.	
μ	Molecular dynamic viscosity.	kg/m.s
ν	Kinematics viscosity.	m^2/s
ω	Vorticity.	
ψ	Stream function.	
ρ	Air density.	kg/m^3
θ	Dimensionless temperature.	

Subscript Symbols

(i,j)	Grid nodes in (r,z) direction
i	Inner annuli
o	Outer annuli

1. Introduction :

Natural convection heat transfer in a cylindrical annulus has attracted much attention with relation to thermal storage systems, solar collectors, spent nuclear air fuel cooling, nuclear reactors, aircraft fuselages insulation, cooling of electrical equipment.

Natural convection in annular enclosure has been studied for many years. Some of studies made a study on laminar natural convection in rectangular enclosure, can be shown as follows:

J. Mizushima and et.al, [2001]^[1], theoretical investigated for transitions of natural convection in an annulus between horizontal concentric cylinders by assuming two-dimensional and incompressible flow field. It was assumed that the inner cylinder is kept at a higher temperature than the outer cylinder. It was confirmed by numerical simulations that dual stable steady solutions exist for Rayleigh numbers larger than a critical value. Diagrams of the steady solutions are obtained by Newton Raphson's method for various values of ratio of the inner cylinder diameter to the gap width and their linear stability was investigated. E. Ramos and et.al, [2004]^[2], analyzed the temperature field generated by an oscillatory boundary layer flow in the presence of a wall with a sinusoidal temperature distribution. The analytical solutions only consider long time behavior when the temperature fields oscillate with the frequency of the flow. The structure of the equation that governs the temperature correction due to convection is similar to that of diffusive waves with the solution consisting of traveling orstanding waves. The temperature distribution is also solved numerically which allows a description of the transient and periodic temperature fields. At short times, the solution has similarities with the traveling waves, while at long times the solution evolves toward a standing wave. As the amplitude of the solution is increased, beyond the linear approximation, the temperature oscillation remains periodic, but more Fourier modes are incorporated. They find that in all cases, the long-time, time averaged heat transfer from the boundary to the fluid is zero. S. M. El-Sherbiny and Atef R. Moussa, [2004]^[3], investigated numerically natural convection in air between two infinite horizontal concentric cylinders at different constant temperatures. The studied covered a wide range of the Rayleigh number, Ra from 10^2 to 10^6 , and the Radius Ratio, (RR) was changed between 1.25 and 10. The differential governing equations were solved using a finite difference method. The flow starts in the conduction regime at low Rayleigh numbers ($Ra = 10^2$) and low radius ratios. It changes to the laminar boundary layer regime as the Rayleigh number or the radius ratio was increased. The study showed that the average Nusselt number increased with the increase of each of Ra and RR in the laminar boundary layer regime. For any fixed value of Ra , the laminar convection starts earlier at higher values of the radius ratio. The annular gap will act as a single inner cylinder in an infinite medium at $RR= 10$ for $Ra = 10^5$. For $Ra > 10^5$, the radius ratio has to be increased much over $RR=10$ in order for the annular gap to behave as a

single cylinder. The numerical results were correlated as a function of Ra and RR.D. Alshahrani and O. Zeitoun, [2005]^[4], investigated numerically natural convection in horizontal cylindrical annular using finite element technique together with SIMPLER algorithm. Annulus geometric configurations of $Do/Di = 2, 3, 4$ and 5 were investigated. The data of heat transfer, represented by Nusselt number Nu_i and effective thermal conductivity ratio ke/k were presented versus Rayleigh number Rai . The data of effective thermal conductivity ratio ke/k were presented versus a new modified Rayleigh number Ram . Three regimes of heat transfer were observed; conduction dominated, transition and convection dominated regimes. The data of heat transfer, represented by the effective thermal conductivity ratio ke/k were correlated and comparison with existed experimental data. A. K. Hassan and J.M. A. Al-lateef, [2007]^[5], presented numerical solutions for the transient natural convection heat transfer problem in horizontal isothermal cylindrical annular enclosed in heated inner and cooled outer cylinders. Solutions for laminar case were obtained within Grashof number based on the inner diameter which varied from 1×10^2 to 1×10^5 in air. The structure of fluid flow such as a velocity vector and temperature distribution as well as Nusselt number were obtained and the effect of diameter ratio on them was examined. The Grashof number was changed with the influence of variation in Prandtl number and diameter ratio. The numerical calculation are summarized by Nusselt number and Grashof number curves with diameter ratios and Prandtl as a parameter, which serves as a guide to natural convection heat transfer calculated from annulus. R.Y. Sakr and et.al, [2008]^[6], investigated Experimental and numerical studies for natural convection in two dimensional region formed by constant flux heat horizontal elliptic tube concentrically located in a larger, isothermally cooled horizontal cylinder. Both ends of the annulus are closed. Experiments carried out for Rayleigh number based on the equivalent annulus gap length ranges from 1.12×10^7 up to 4.92×10^7 , the elliptic tube orientation angle, θ , varies from 0° to 90° and the hydraulic radius ratio, HRR, of 6.4 . These experiments were carried out for axis ratio of elliptic tube (minor/major= b/c) of $1:3$. The numerical simulation for the problem is carried out by using commercial CFD code. The effects of the primary objective of this research is to evaluate the usefulness of computational fluid dynamic techniques in modeling laminar natural convection in vertical annular enclosure with linearly varying wall temperature as following

- The solution procedure is solving the governing equations (continuity, momentum and energy) equations after applying the assumptions: laminar flow, Boussinesq approximation is used, axisymmetric two-dimensional flow, no heat generation and radiation and conduction effects are neglected.

- After that these equations will be converted to dimensionless form then the $(\psi-\omega)$ scheme will be applied on these equations and it will be solved numerically by computational fluid dynamics technique by using finite difference method.
- The transfer equations are solved using an explicit finite difference scheme and the Gauss-Seidel algorithm. The numerical results, including the evolutions of velocity, temperature, as well as the Nusselt number are presented.

The results will be in form of contours (stream function, vorticity and isotherm) and a plot between Nusselt orientation angle as well as other parameters such as elliptic cylinder axis ratio and hydraulic radius ratio on the flow and heat transfer characteristics were investigated numerically. The numerical simulations covered a range of elliptic tube axis ratio from 0.1 to 0.98 and for hydraulic radius ratio from 1.5 to 6.4. The results showed that the average Nusselt number increases as the orientation angle of the elliptic cylinder increases from 0o to 90o and with Rayleigh number as well.

1.1 Objective

The primary objective of this research is to evaluate the usefulness of computational fluid dynamic techniques in modeling laminar natural convection in vertical annular enclosure with linearly varying wall temperature as following

- § The solution procedure is solving the governing equations (continuity, momentum and energy) equations after applying the assumptions: laminar flow, Boussinesq approximation is used, axisymmetric two-dimensional flow, no heat generation and radiation and conduction effects are neglected.
- § After that these equations will be converted to dimensionless form then the $(\psi-\omega)$ scheme will be applied on these equations and it will be solved numerically by computational fluid dynamics technique by using finite difference method.
- § The transfer equations are solved using an explicit finite difference scheme and the Gauss-Seidel algorithm. The numerical results, including the evolutions of velocity, temperature, as well as the Nusselt number are presented.
- § The results will be in form of contours (stream function, vorticity and isotherm) and a plot between Nusselt number versus Rayleigh number will be done.
- § Comparisons with other investigations will be introduced to make sure from the present work results.

After reviewing the pervious researches, it is obvious that the study of free convection heat transfer of air in vertical annular enclosure, study for $Pr=0.71$ and Ra from $(10^3$ to $10^5)$

with linear boundary condition for temperature, the effect of aspect ratio increasing on the same previous cases will be study.

2. Mathematical Model :

A schematic representation of the system under investigation and the proper cylindrical coordinates system (r,z) is shown in **Figure(1)**.

The aim of the present study is to investigate natural convection in two dimensional annular enclosure filled by air (Pr=0.71).

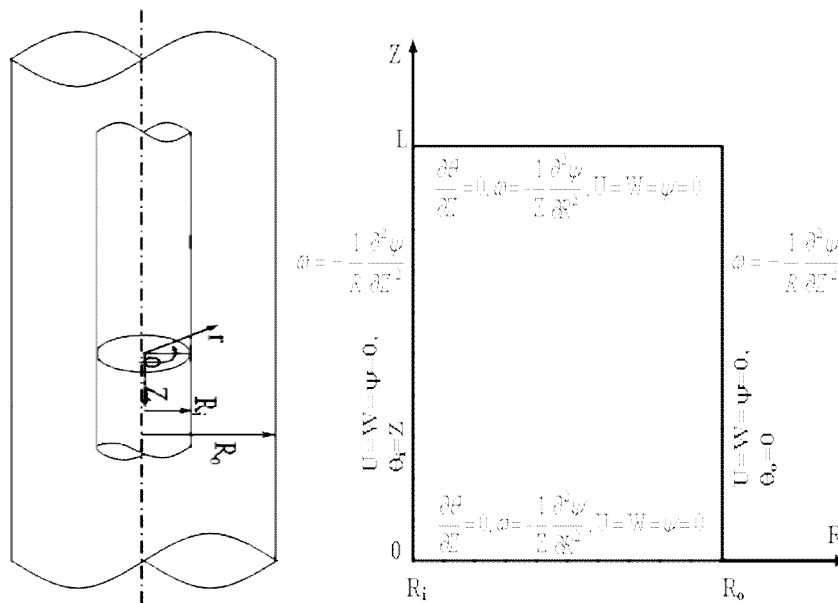


Fig. (1): Physical domain with boundary conditions.

The governing equations are transformed into dimensionless forms under the following non-dimensional variables (**Schwab and De Witt,[1970]^[7]**):

$$\begin{aligned} (R = r/(r_o - r_i)), (Z = z/(r_o - r_i)), (U = u(r_o - r_i)/a), (w = w(r_o - r_i)/a), \\ (P = p(r_o - r_i)^2/ra^2), (q = (T - T_o)/(T_i - T_o)), (Pr = (\eta/a)), \\ (Ra = (Pr gb(T_i - T_o)(r_o - r_i)^3)/\eta^2), (AR = L/(R_o - R_i)). \end{aligned}$$

In terms of these variables, the stream function, vorticity and energy equations respectively becomes

Dimensionless equation of vorticity definition as:

$$w = \frac{\partial W}{\partial R} - \frac{\partial U}{\partial Z} \tag{1}$$

The vorticity transport equation is:

$$\frac{\partial(Uw)}{\partial R} + \frac{\partial(Ww)}{\partial Z} = Ra Pr \frac{\partial q}{\partial R} + Pr \left[\frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial(Rw)}{\partial R} \right) + \frac{\partial^2 w}{\partial Z^2} \right] \quad (2)$$

The radial and vertical velocities can be written as follows respectively:

$$U = \frac{1}{R} \frac{\partial y}{\partial Z}, W = -\frac{1}{R} \frac{\partial y}{\partial R} \quad (3)$$

Stream function equation resulted for vorticity as:

$$-w = \frac{1}{R} \left(\frac{\partial^2 y}{\partial R^2} - \frac{1}{R} \frac{\partial y}{\partial R} + \frac{\partial^2 y}{\partial Z^2} \right) \quad (4)$$

The dimensionless energy equation can be transformed to another form as follows:

$$U = \frac{1}{R} \frac{\partial y}{\partial Z}, W = -\frac{1}{R} \frac{\partial y}{\partial R} \quad (5)$$

$$\frac{1}{R} \frac{\partial(RUq)}{\partial R} + \frac{\partial(Wq)}{\partial Z} = \left[\left(\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial q}{\partial R} \right) \right) + \frac{\partial^2 w}{\partial Z^2} \right] \quad (6)$$

$$\frac{1}{R} \frac{\partial}{\partial R} \left(\frac{\partial y}{\partial Z} q \right) + \frac{\partial}{\partial Z} \left(-\frac{1}{R} \frac{\partial y}{\partial R} q \right) = \left[\left(\frac{1}{R} \frac{\partial}{\partial R} \left(R \frac{\partial q}{\partial R} \right) \right) + \frac{\partial^2 w}{\partial Z^2} \right] \quad (7)$$

2.1 Boundary Conditions :

The boundary conditions of velocity and temperature fields are shown in **Fig(1)** presented as (Kumar, [1988]^[8] and Oosthuizen and Naylor, [1999]^[9]:

$$\left. \begin{aligned} R = R_i : U = W = \Psi = 0, q = Z, -\Omega &= \frac{1}{R} \frac{\partial^2 \Psi}{\partial Z^2} \\ R = R_o : U = W = \Psi = 0, q = 0, -\Omega &= \frac{1}{R} \frac{\partial^2 \Psi}{\partial Z^2} \\ Z = 0 : U = W = \Psi = 0, \frac{\partial q}{\partial Z} = 0, -\Omega &= \frac{1}{Z} \frac{\partial^2 \Psi}{\partial R^2} \\ Z = 1 : Y = 0 : U = W = \Psi = 0, \frac{\partial q}{\partial Z} = 0, -\Omega &= \frac{1}{Z} \frac{\partial^2 \Psi}{\partial R^2} \end{aligned} \right\} \quad (8)$$

2.2 Nusselt Number Calculation :

The local heat transfer rate along the heated wall is obtained from the heat balance that gives an expression for local Nusselt number as (Oosthuizen and Naylor, [1999]^[9]):

$$Nu = -\frac{\partial q}{\partial R} \Big|_{R=R_i} ; 0 \leq Z \leq 1 \quad (9)$$

In difference form this can be written as,(A. M. Salman,[2003]^[10]):

$$Nu = \frac{25q_{1,j} - 48q_{2,j} + 36q_{3,j} - 16q_{4,j} + 3q_{5,j}}{12\Delta R} \quad (10)$$

The average value for Nusselt number is defined by

$$\overline{Nu} = \frac{1}{L} \int_0^L Nu dZ \quad (11)$$

This equation can be readily evaluated using Simpson's rule.

3. Numerical Solution:

The governing equations will be solved numerically by using explicit finite difference method (Tannehill et al. (1997))^[11]. The domain is divided, the node-point has subscripts (i,j) denoting cylindrical coordinate in (r, z) directions. The coordinates for each node ($R_{i,j}=R_i+i\Delta R$), ($Z_{i,j}=j\Delta Z$), where $i=0,1,2,3,\dots,n$; and $j=0,1,2,3,\dots,m$, as shown in **Figure(2)**.

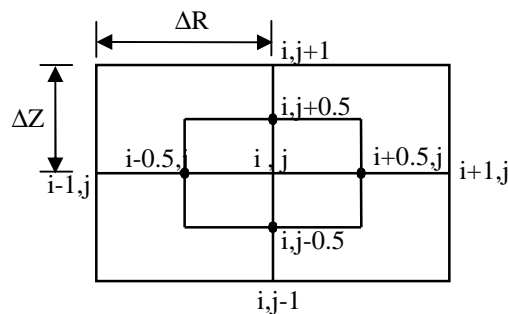


Fig. (2) Nodal points.

The governing dimensionless differential equations are discretized to a finitedifference form is used. The computational scheme, based on Successive Over Relaxation, SOR, is arranged to solve the three equations for the n^{th} iteration step. The initial values over the field for θ , ω and ψ are assumed zero to all internal nodes are taken as initial starting values. Over-relaxation is actually used so the "updated" values of $\phi_{i,j}$ are actually taken as:

$$f_{i,j}^{new} = f_{i,j}^{old} + r(f_{i,j}^{calculated} - f_{i,j}^{old}) \quad (12)$$

Where the subscripts i and j refer to a grid node, ϕ is a general dependent variable (θ, ω , or ψ). The relaxation parameters, $\gamma\theta = \gamma\omega = 1$ and $\gamma\psi = 1.6$ give stable numerical computation $Ra \leq 10^4$.

The criterion for convergence is examined according to a realistic condition for each state variable at each node as:

$$\frac{|f_{i,j}^{new} - f_{i,j}^{old}|}{|f_{i,j}^{old}|} \leq E_{max} \quad (13)$$

Where the subscripts i and j refer to a grid node, ϕ is a general dependent variable (θ, ω , or ψ) and E_{max} is a small quantity of error set to 10^{-5} for $Ra \leq 10^5$.

A computer program in (Fortran 90) was built to execute the numerical algorithm which is mentioned above; it is general for a natural convection in two-dimensional annular enclosure, as shown in the flow chart **Figure(3)**.

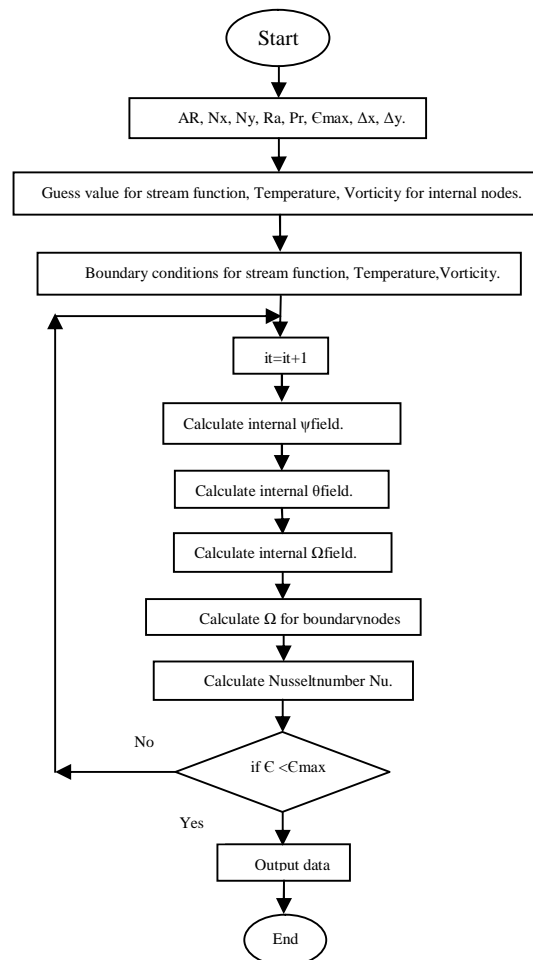


Fig.(3) Flow chart for computer program.

4. Result and Discussions:

In this section they obtained results will be discussed. all the results are calculated for air ($Pr=0.71$), while Ra numbers within ($10^3 \leq Ra \leq 10^5$), after that a comparison will be made with previous researches.

Figure(4) show the contours of streamlines for different Ra numbers ($10^3 \leq Ra \leq 10^5$) and aspect ratio ($AR=1$).The mechanism of flow occurs when the air near hot wall is heated causing increasing density and the air will be start to move near the hot wall towards the cold wall. Also showed streamlines for ($Ra=10^3, 10^4$) will be almost similar in Figures (2a, 2b) except that in $Ra=10^5$, the streamlines will be closer to the wall, this means that the buoyancy force increase causes an accelerate flow.

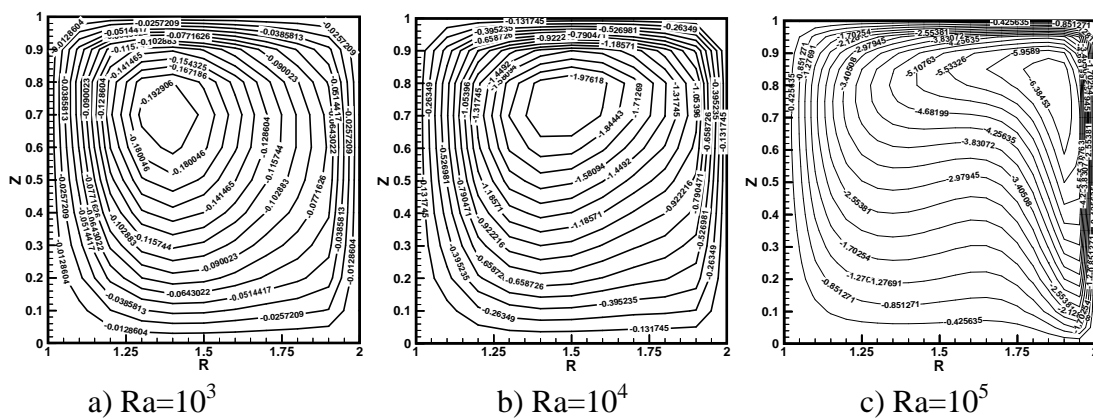


Fig.(4) Stream lines contour for AR=1.0

Figure(5) Show the stream function contours for Ra numbers ranging within ($10^3 \leq Ra \leq 10^5$) at $AR=2.0$.

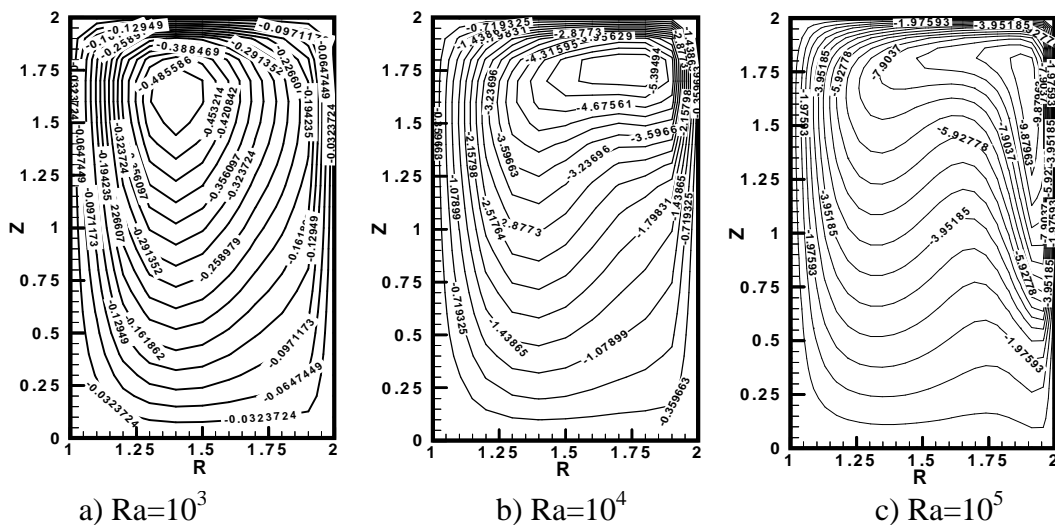


Fig. (5): Stream lines contour for AR=2.0

The stream lines will be closer to the cold wall, this means that buoyancy force increase cause an accelerate to flow, Also shown a bicellular flow in stream lines contour because the velocity nearby cold wall is larger than those which are far away from cold wall. The same behavior approximately in **Figure(6)**, Also shown a bicellular flow clearly at $Ra=10^5$ and $AR=3$.

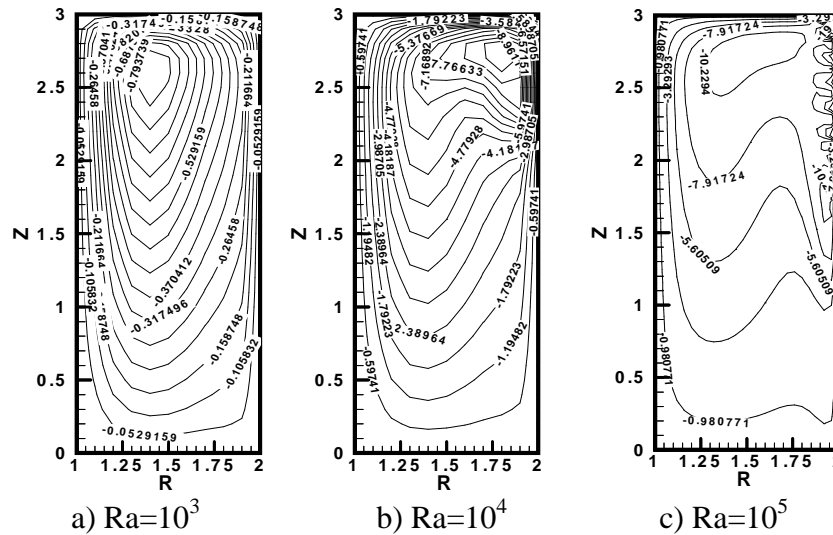


Fig. (6): Stream lines contour for $AR=3.0$

Figs. (7), (8) and (9) represent the effect of Ra number on the steady-state variation for $Ra = 10^3-10^5$ and aspect ratio (AR) which ranging within (1.0 to 3.0) by isotherms lines contours. Heat transfer by conduction which extended; it considered to be start of convection region. Also single cellular flow regime appears

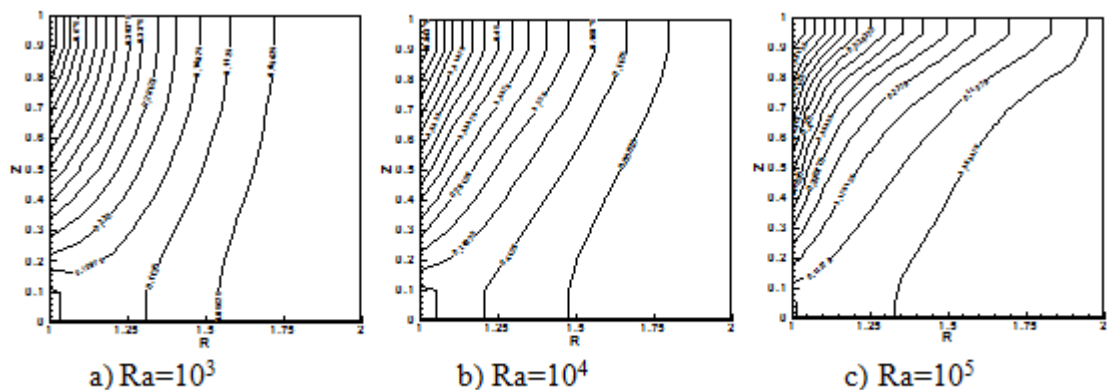


Fig.(7) Isotherm lines contour for $AR=1.0$

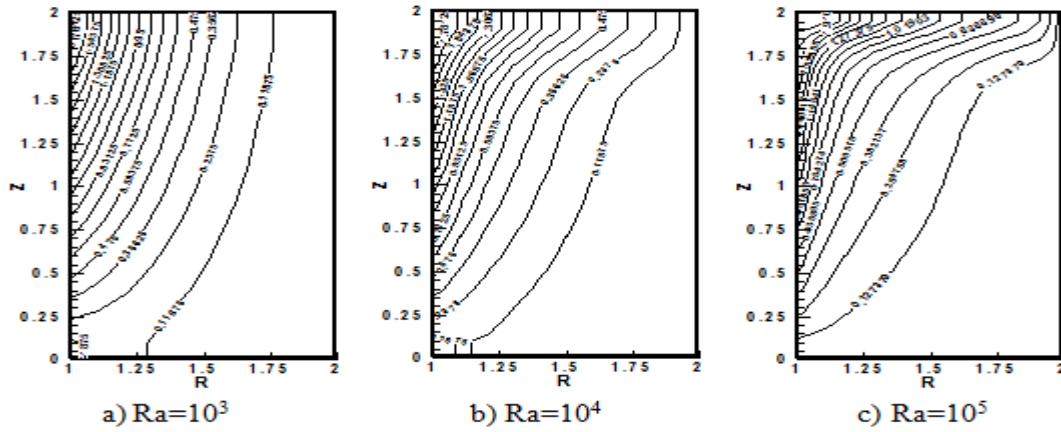


Fig.(8) Isotherm lines contour for AR=2.0

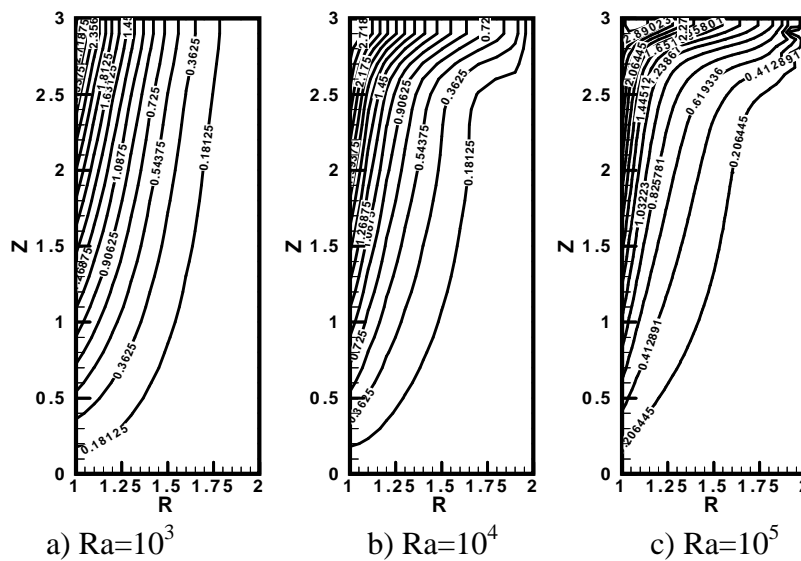


Fig. (9): Isotherm lines contour for AR=3.0

Figures. (10), (11) and (12) show the distribution of velocity vector for different Ra numbers ($103 \leq Ra \leq 105$) at AR=1,2,3 respectively, they showed that the hot fluid move along hot side and before move downwards along the cold side wall.

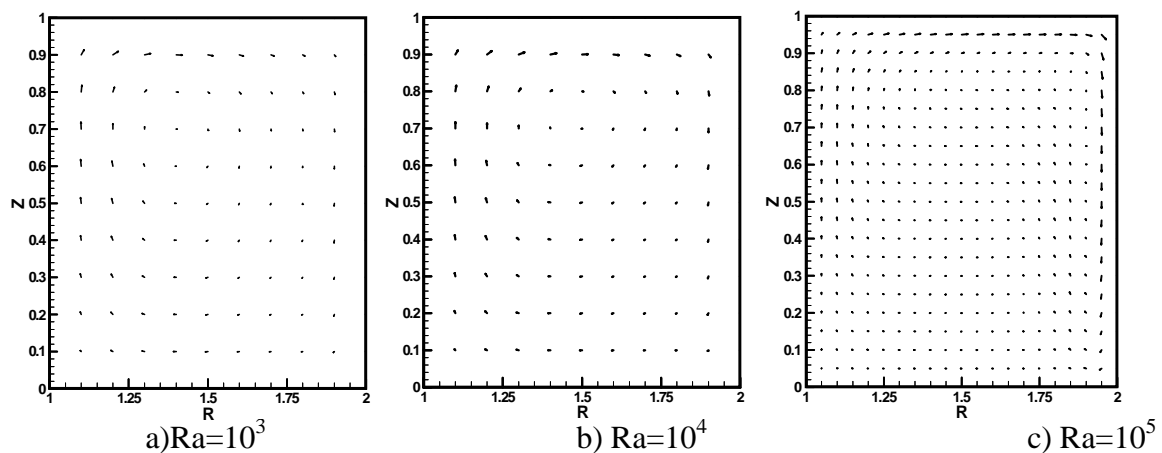


Fig. (10) Velocity vector for AR=1.0

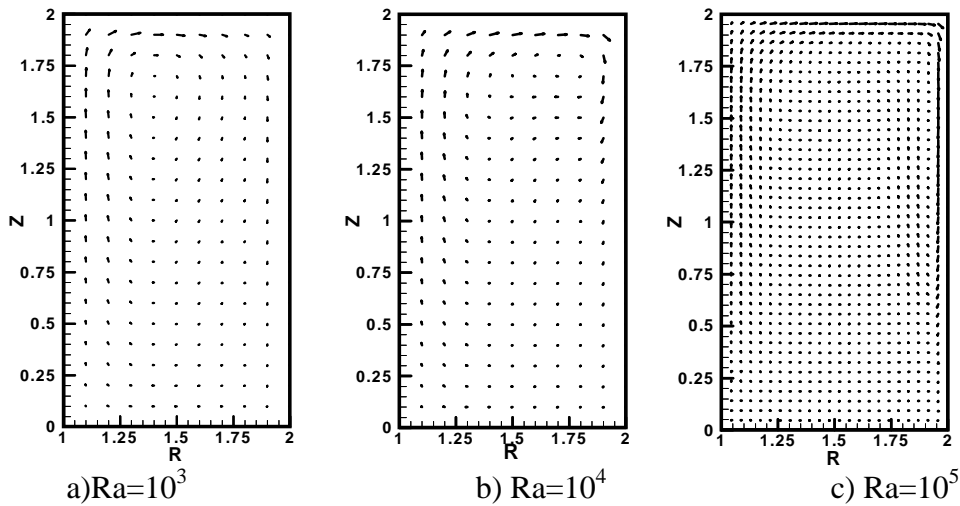


Fig.(11) Velocity vector for AR=2.0

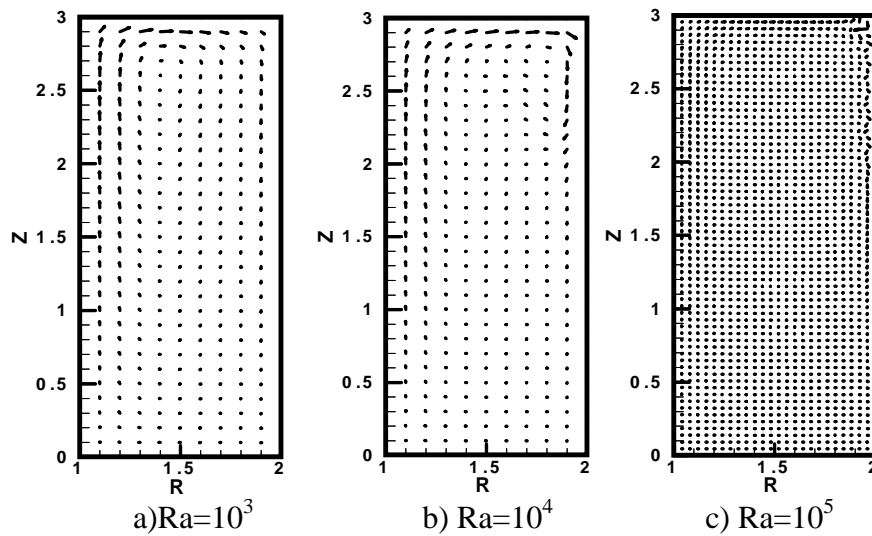


Fig.(12) Velocity vector for AR=3.0

Figure(13) represent the relation between average Nusselt number and Rayleigh number for aspect ratio (1.0 - 3.0), average Nusselt number increase with increased aspect ratio because the area of heat transfer will be increased. And the same behavior in **Figure.(14)** shows the relation between average Nusselt number and aspect ratio for Ra ranging with (10^3 - 10^5).

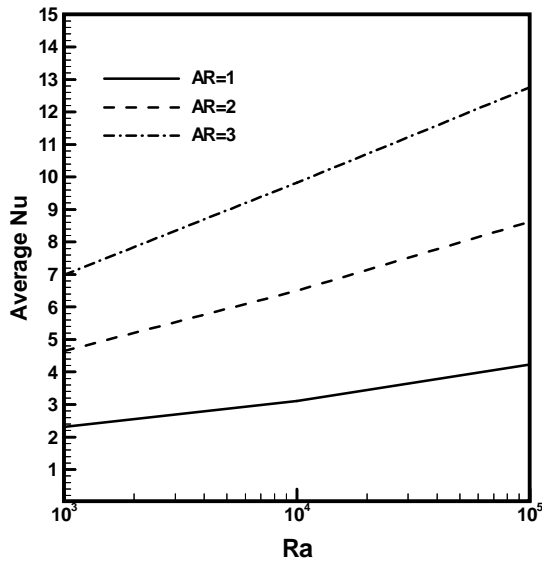


Fig.(13) Variation of average Nusselt no. with Ra no.

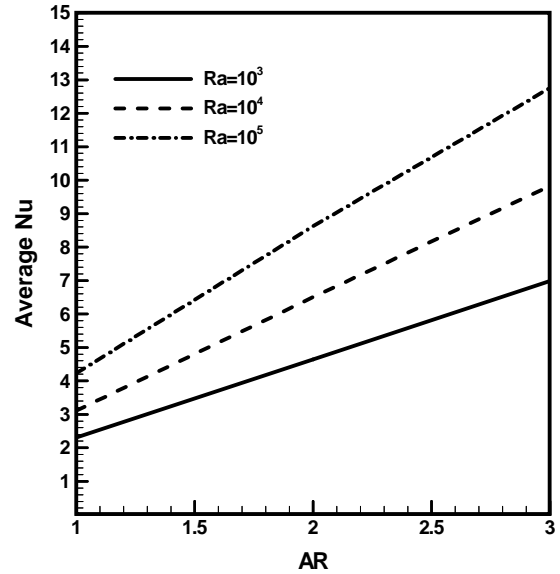


Fig.(14) Variation of average Nusselt no. with AR.

Figure(15) show the comparison of average Nusselt number for present study which is calculated numerically with results mentioned in reference of (Kubair and Simha, 1982)^[12] at aspect ratio (AR=1.0). The agreement is good and which validate the present computational model.

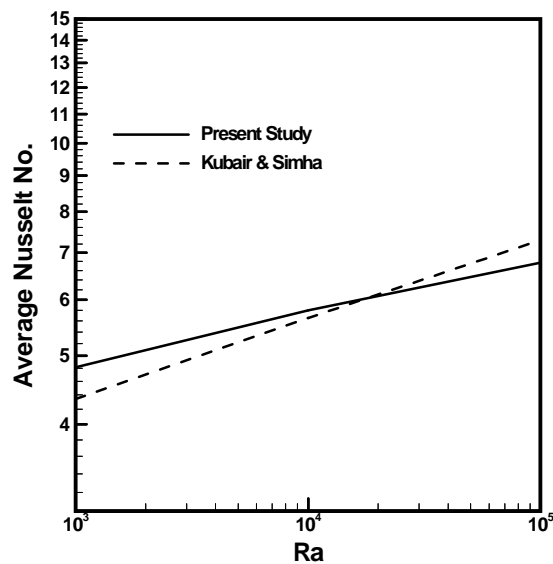


Fig.(15) Comparison between present study and kubair & Simha [10] for AR=1.

5. Conclusions:

In this research natural convection in annular enclosure filled by air ($Pr=0.71$) is studied. The solution scheme is validated by comparison with last research, where the agreement was good. The heat transfer operation by convection for ($Ra \geq 10^3$). The streamlines shows a single cell form except at ($Ra \geq 10^5$) where it shows a bicellular form. Heat transfer is increased with increasing the value of aspect ratio. The variation of average Nusselt number with aspect ratio and Rayleigh number. There is a remarkable increase in the average Nusselt number with an increase in the aspect ratio.

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