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Experimental Determination Of Elastic Constant Of Composite Materials Using Vibration Properties

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Abstract :

In this study, a correlation between an approximate analysis of beam vibration problem and experimental measurements of vibration property was a achieved in an attempt to develop a simple and reliable experimental method for determining the elastic constant of materials as a non-destructive test.

The analytical solution was based on Euler and Rayleigh beam models with simply supported boundary conditions, and the experimental work was based on the Dunkerly's approach. A development of an existing test setup was done to be more suitable for beam specimen of composite materials.

The present experimental measurements of the elastic constant were compared with published results and it is found that the proposed experimental work is provided an acceptable and reliable measurements.

The comparisons indicate errors in the measurements that are ranging between (3.1-12.7) % based on Euler model and (0.87-4.6) % based on Rayleigh model.

Finally, a numerical solution of the problem based on the feedback of the present experimental results was developed using a computer package program called ANSYS.

The numerical results reflected that the proposed experimental scheme is a simple, reliable and accurate one.

الحساب العملي لمعامل مرونة المواد المركبة باستخدام خواص الاهتزازات

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الخلاصة :

في هذا البحث تم إيجاد طريقة توافقية بين التحليل النظري التقريبي والقياس العملي لخواص الاهتزازات لغرض ايجاد طريقة عملية مبسطة ومعتمدة لغرض حساب معامل مرونة المواد المركبة كإحدى الطرق التي تسمى بالاختبار اللااتلافي.

تم اعتماد طريقتي اويلر ورايلي لاشتقاق معادلات الاهتزاز الطولي لعينة عتبة مثبتة بطريقة التعليق الحر واعتماد طريقة دنكلريز لحسابات الجانب العملي. تم تطوير الجهاز ألمختبري ليكون أكثر ملائمة لعينات المواد المركبة. وقد أوضحت النتائج وجود تطابق في القياس النظري والعملي للعينات وبنسبة خطاء تتراوح بين (12.7-3.1) % باعتماد طريق اويلر و (4.6-0.87) % باعتماد طريقة رايلي.

تم استخدام النتائج من الطريقة أعلاه كبيانات لبرنامج تم أعداه باستخدام برنامج حاسوبي يسمى (ANSYS) لغرض التحقق والمقارنة من نجاح الطريقة المقترحة والتي أكدت النتائج على أنها طريقة مبسطة ومعتمدة لحساب معامل مرونة المواد المركبة.

Symbol	**Meaning**
A*	cross-sectional area (m^2)
a^2	wave number
E_{RAY}	elastic constants using Rayleigh models
E_{EUL}	elastic constants using Euler-Bernoulli models
I*	beam second area moment (m^4)
L*	length of the beam (m)
M	mass of beam (kg).
m_{beam}	mass of the beam specimen(kg)
$m_{exciter}$	mass of the exciter motor assembly(kg).
ω_{beam}	frequency of beam specimen (rad/sec).
ω_s	fundamental frequency of beam specimen plus exciter
$\omega_{exciter}$	fundamental frequency of exciter mounted on the beam specimen
ρ*	beam material density (kg/m^3)

1. INTRODUCTION

A wide variety of materials are used in the design and building of the structure and objects. The process of mechanical design is highly dependent on the strengths and bending properties of the material being used. These factors are in turn dependent on the elastic properties such as Young's modulus. Hence the knowledge of the Young's modulus is very much vital for design. There have been many methods for measurement of the Young's modulus. One of these methods is the tensile test, which represents a destructive test of material. But the present research proposed scheme which can be described as a one of non – destructive type of material testing.

Many researchers were presented different work related to the measurements of elastic constant using non–destructive type of tests. R. Lance Willis et. al. [1] determined the young and shear dynamic modulus of visco-elastic materials from laser vibrometric measurements of the surface motion of a three-dimensional sample excited by a piezoelectric actuator inside a chamber with controllable temperature and static pressure. The modulus are estimated from an inversion code that minimizes the difference between the data and the predictions from a finite element model in which the elastic modulus are the adjustable parameters. The technique is first used to measure the dynamic properties of homogeneous samples and the results are compared with those obtained by the standard rod resonance technique.

Lauren M. Pederson [2] presented the preliminary results from an experiment performed to determine the temperature dependence of the modulus of elasticity for a thermoplastic iso grid tube. To do this, the iso grid tube was subjected to axial tensile loads of (0-100) lbf and strain was measured at room and at elevated temperatures (from 75-200 °F) using two protocols. Thus, the purpose of his research is to determine the modulus of elasticity of the tube as a function of temperature, with particular emphasis on measurements near the glass transition temperature of the thermoplastic matrix.

Chi Hsiang Pan [3] used a novel method for determining young's modulus of thin films with compact micro machined test structures and without using any extra load. The test structures comprise of a pair of micro-strain gauges and a cantilever beam. An analytical model is derived to extract the young's modulus of test structures. The obtained young's modulus is reduced a little as residual stress increases.

H. Kinney et al.[4] used the technique of resonant ultrasound spectroscopy (RUS) to measure the second-order elastic constants of hydrated human dentin. Specimens were placed between two transducers, and the resonant frequencies of vibration were measured between 0.5 and 1.4 MHZ. The magnitudes of the elastic constants determined with RUS are in good agreement with values determined from sound speed measurements.

S. Siva Shashidhara et al. [5] compared between two goniometry based immersion techniques for the measurement of elastic constants in isotropic and transversely isotropic materials. Measurements were carried out on transversely isotropic unidirectional (glass-epoxy and graphite-epoxy) composite materials. From the measured velocity data, the elastic constants were determined through a numerical inversion. These techniques (through-transmission and back-reflection) were verified using contact testing, mechanical testing, rule of mixtures estimation and the data provided by the manufacturers.

Han Park et. al. [6] adopted the non-linear least squares method (NLLS) to understand visco-elastic properties of composite materials for design and analysis of the structures and to develop finite element codes for a composite structure with several damping materials. In this study, an advanced technique for obtaining accurate loss factor and young's modulus of a composite structure is introduced based on a multi degree of freedom curve-fitting method. The loss factor and young's modulus of a composite structure are measured for different temperatures by performing the test in a vibration measurement room where temperature varies from 5 C^o to 45 C^o.

T. Pramila et. al. [7], determined the elastic constants of aluminum from the analysis of laser generated ultrasonic bulk waves. A pulsed Nd:YAG laser (1064 nm) is used for ultrasonic generation in a thick stepped Al sample and a He-Ne laser is used for heterodyne detection of the generated signals. Their results show the applicability of the study of laser generated bulk waves for the determination of elastic constants of any bulk material.

Yu-Hua Lin and Chia-Lung [8] presented an inverse method to derive the elastic constant of thick composite plates of AS4/PEKK material from the resonance frequencies of a free–edge test specimen based on modal vibration test. A mixed numerical experimental identification procedure is used. The optimization technique, Hybrids genetic /Simulated/Annealing algorithm, has been used. The results show that different stacking sequences and numbers of frequencies have effect on the determination of the elastic constants.

G V Smurthy, and P. Nikhat [9] studied the behavior of elastic constants and the variation on heat treatment in a nickel base super alloy Nimonic 263 by ultrasonic velocity measurements. The results indicated that the elastic moduli of the material are very sensitive to any minor compositional changes, resulting due to the formation of intermetallic phases on heat treatment and can be effectively monitored by ultrasonic.

2. THEORETICAL FORMULATION

It is known that the problem of transversely vibrating beam represents a complicated situation, so that, there is no paper that presents the complete solution from the formulation of the governing differential to its solution, taken into consideration all beam nonlinearities. Thus, for comparison purpose, two different models of beam named, Euler-Bernoulli and Rayleigh are adopted. The mathematical formulation of the problem of beam transfers vibration with simply supported boundary conditions can be followed in details in reference [10] and the frequency – elastic constant relationship using Euler-Bernoulli and Rayleigh models are given by:-

$$E_{EUL} = \left(\frac{w_{BEAM}}{a^2}\right)^2 \left(\frac{r^* A^* L^{*4}}{I^*}\right) \qquad (1)$$

$$E_{RAY} = \left(\frac{w_{BEAM}}{p^2}\right)^2 \left(\frac{ML^{*3}}{I^*}\right) \qquad (2)$$

3. EXPERIMENTAL WORK

As was mentioned previously that the present work is aimed to introduce a simple and reliable experimental procedure for determining the elastic constant of the material, thus in this section the experimental work is given great attention. Firstly it is important to note that the materials of specimens used are metallic and nonmetallic materials (composite materials). The metallic material elastic constant are well known, so that, the determination of their elastic constants is used as a verification case studies to prove the reliability of the proposed experimental method presented by the present work.

The experimental work was carried out at mechanical engineering laboratories /mechanical engineering department / college of engineering / Kufa University. The experimental is achieved by what is called the universal vibration apparatus with manufacturing of additional jaws for clamping of the composite material specimens. The test setup is shown in **Figure.(1).**

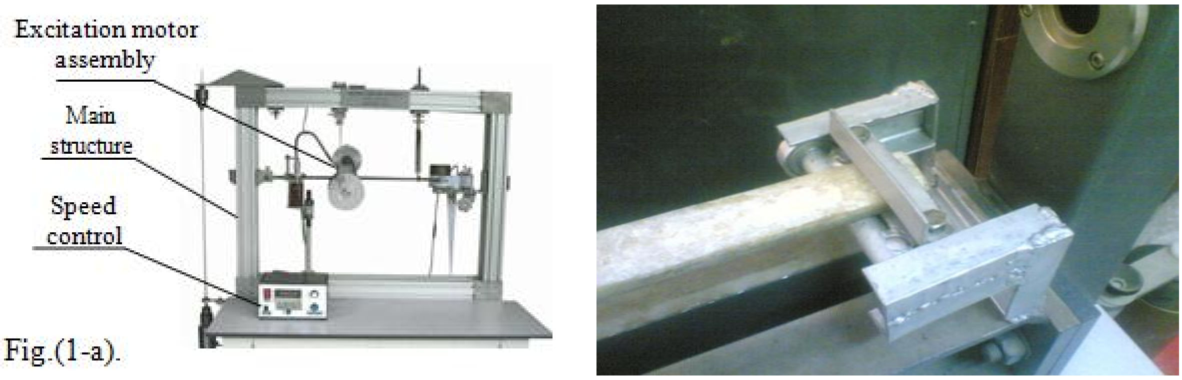

Fig.(1-a).

Fig.(1-b).

Fig.(1).Experimental test apparatus, (a)experimental setup, (b) Manufactured fixtures (jaws) for specimens of composite materials.

The composite materials consist of a chopped strand mat (CSM) fiber, and polyester resin as a matrix. The fibers of the composite specimens were distributed in a cross-angle form to produce three different specimens. The specimens are manufactured in a shape of beam of rectangular section according to the recommendations of the testing apparatus manufacturer [11]. The testing material specifications are shown in **Table (1)** below:-

Table (1):- Specimens properties.

No	Material	Mass (kg)	Specimens dimensions LxWxH (mm)
1	Steel	2.095	838 x 25 x10
2	Aluminum	0.655	838x 30x 10
3	Copper	2.205	838x 30 x 10
4	Composite material specimen 1	0.35	838x 30 x10
5	Composite material specimen 2	0.365	838x 30 x10
6	Composite material specimen 3	0.37	838x 30 x10

It is worthy to mention that all measuring instruments used in recording of data are calibrated using standard calibration tools. Such as the weighting scale for measuring of the specimen masses was calibrated with standard masses. Also, the speed of the harmonic excitation measurements unit was calibrated using stroboscopic device.

The experimental procedure including of recording of natural frequency of the beam specimens shown in **Table (1)** above by using the universal test apparatus. Each of these specimens was suspended in a simply supported end conditions and motor assembly with unbalanced disk was rotated in order to produce a harmonic excitation force. In order to obtain the beam specimen natural frequency the motor speed was controlled by speed control unit and it is increased gradually until the resonance condition is attained. The resonance reading was recorded in increasing and decreasing modes. It is important to explain that masses with specific values are added to the motor assembly in each test and then the resonance condition for this test is recorded. This is have been done in an attempt to obtain the resonance condition in more accurate readings, because the experimental procedure adopted in the present work is greatly depending on the measurements of the natural frequency at resonance condition. The experimental readings of the natural frequency using resonance conditions with added masses are shown graphically in **Figures. (2) to (7).**

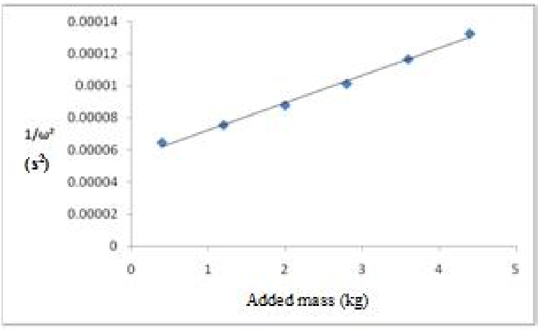

Fig. (2) Shows the natural frequency of the structure against the added mass for steel specimen.

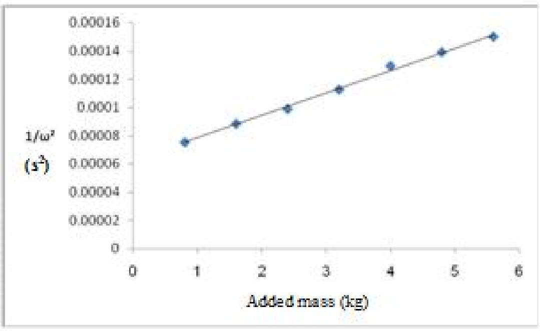

Fig. (5) Shows the natural frequency for the structure against the added mass for composite material specimen No.1.

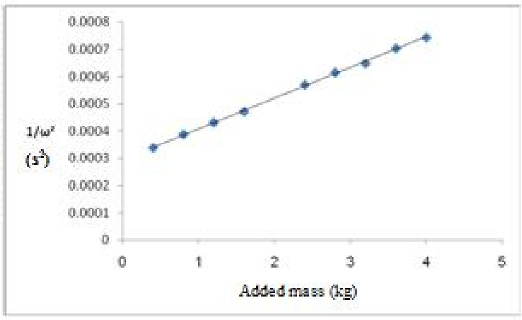

Fig. (3) Shows the natural frequency of the structure against the added mass for aluminum specimen

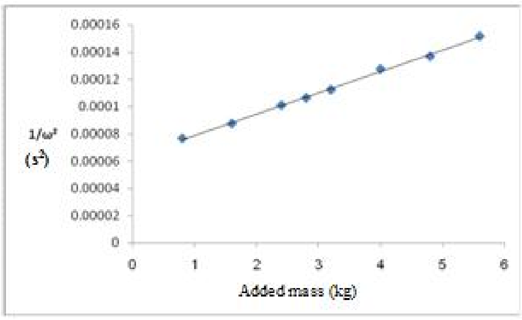

Fig. (6) Shows the natural frequency of the structure against the added mass for composite material specimen No.2.

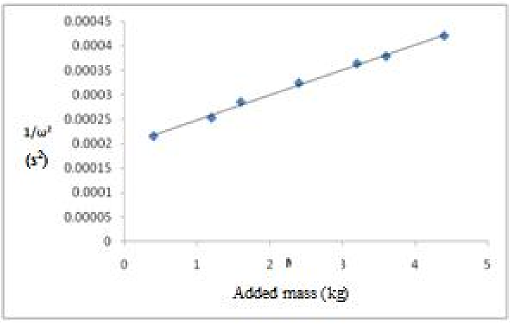

Fig. (4) Shows the natural frequency of the structure against the added mass for copper specimen.

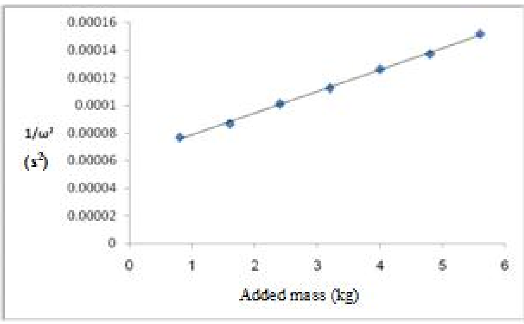

Fig. (7) Shows the natural frequency of the structure against the added mass for composite material specimen No.3.

4. RESULTS AND DISCUSIONS

In mechanical engineering, it is known that a specified structure can be allowed to vibrate under a forced vibration condition until the resonance condition is satisfied (over a short period of time). At the resonance condition it is known that the ratio ($\omega_{force}/\omega_{nat}$) is equal to one. Thus, the measurement of the forced frequency becomes a measure for the natural frequency. But the problem appeared in the experimental measurements of the natural frequency that they are represents the measurements of the natural frequency of the beam and the vibrating structure (added masses plus excitation motor). Thus for overcoming this problem, the Dunkerley's method is incorporated in the present experimental work.

Dunkerley's formula, show that, the fundamental frequency of the beam by itself may be determined by the following relationship **[12]**:-

$$\frac{1}{w_s^2} = \frac{1}{w_{beam}^2} + \frac{1}{w_{exciter}^2} \qquad (3)$$

The effect of the added mass on the beam specimen natural frequency has been removed using the results shown in **Figures (2-7),** by extending the line of the graph until which it is cutting the ($1/\omega^2$) axis. Thus, the results of the natural frequency corresponding to this intersection point represent the natural frequency of the beam specimen mass and the exciter motor. The above formula can be formulated in more convenient form take into consideration the effect of the position of the motor exciter along the specimen. This attempt can be done with mathematical manipulations depending on the influence coefficient of the mass of the motor exciter assembly which is considered at the mid span of the specimen , and it is may be given by:-

$$\left(\frac{w_s}{w_{beam}}\right)^2 = \frac{1}{1+2.09\dfrac{m_{exciter}}{m_{beam}}} \qquad (4)$$

Using the formula given in Eq.(5) and experimental measurements of the natural frequency (ωs) obtained from graphs shown in **Figures(2-4),** and using the expressions for the elastic constants given in Eqs.(1) and (2), the experimental measurements of the elastic constant of metallic materials are given in **Table (2).**

Table (2). Experimental measurements of elastic constants of metallic materials.

E	Steel specimen	copper specimen	Aluminum specimen
E $_{table}$ GPa) Ref. [3]	200	110	68
E $_{Euler}$ (GPa)	197.09	102.65	57.216
E $_{Ray}$ (GPa)	189.38	100.938	55.253
% error based on Euler’s model	1.455	6.681	15.858
% error based on Rayleigh's model	5.310	8.238	18.745

The range of the percentage error in these results compared with that obtained from Ref. [13]. It is shown that an error ranges of (1.455-15.8858) % using Euler's model, and of (5.31-18.745) % using Rayleigh's model. These percentage errors are attributed to many reasons, such that, the simplicity of way of measurements of the resonance conditions and may be the differences in the type of the material used in the presents work and that available in Ref [2-10]. In spite of these errors, the results indicated that the experimental measurements method of the elastic constants proposed in the presents study provided reliable results with simplicity of the testing setup and rout of the experimental work. These comparisons are aimed to verifying the reliability and the simplicity of the experimental measurements method adopted in the present work. Because actually, the present work mainly aimed to presents a reliable and simple experimental method to determine the elastic constants of the composite materials experimentally. Accordingly, the same experimental procedure used in determination of the elastic constants of the metallic material can be followed to determine the elastic constants of the composite materials. The results are shown in **Table (3)**.

Table(3). Experimental measurements of elastic constants of composite materials.

Elastic constant	Composite material Specimen 1	Composite material Specimen 2	Composite material Specimen 3
$E_{\text{Theo.}}$ (GPa)	34.30	35.32	36.34
E_{Euler} (GPa)	38.66	37.78	37.47
E_{Ray} (GPa)	34.99	35.01	34.66
% error based on Euler's model	12.711	6.964	3.109
% error based on Rayleigh's model	2.011	0.877	4.623

The theoretical elastic constant of the composite material was calculated on the basis of the rule of mixture (See Ref.[12] for more details).

The investigation of the results indicates that the value of the error is ranging between maximum and minimum values of (3.101-12.711) based on Euler model, and (0.877- 4.623) based on Rayleigh model. Accordingly, it can be concluded that the proposed experimental scheme provided a good and reasonable experimental results of elastic constants of composite materials. Also, the using of vibration properties in determination of elastic constants in a simple manner with the correlation between, experimental and theoretical methods proposed in the present study, may resulting in a powerful method can be a adopted for measuring of elastic constants within an acceptable degree of accuracy and reliability.

For more verification purposes, other case study was adopted in the present work. A computer finite element program was developed (built) using a package which is called (ANSYS). The problem of beam transverse vibration with simply supported end conditions was solved with feeding back of the results of the elastic constant obtained on the basis of the proposed experimental work. So that, the results of the beam natural frequency obtained as a output of the program, was compared with the experimental results of present experimental work. The comparison is shown graphically in **Figure.(8),** and the results indicate again that the present work provided an acceptable measurements of the elastic constant of the composite materials.

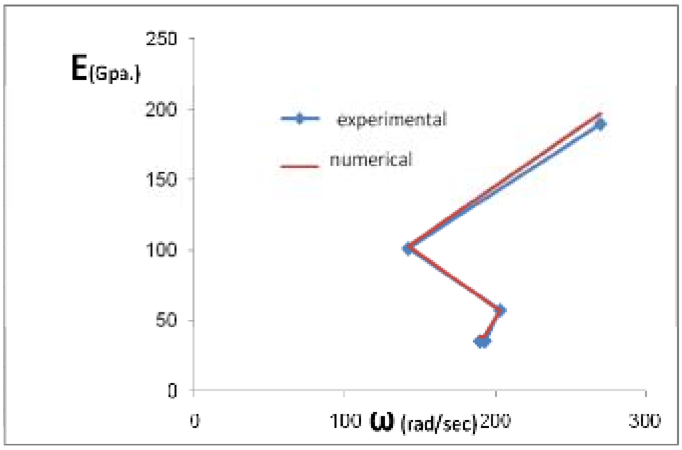

Fig.(8) show the comparison between the experimental and numerical values of the modulus of elasticity.

5. CONCLUSIONS

An attempt was made in the present study to provide a simple and reliable experimental procedure to measure the elastic constant of the composite materials as a nondestructive test. The proposed method was based on the measurements of the vibration properties of a beam specimen vibration problem.

The experimental results indicate that the proposed experimental method provided an acceptable, simple and reliable way in determination of modulus of elasticity of composite materials with acceptable errors ranging between (3.101-12.711) based on Euler's model, and (0.877- 4.623) based on Rayleigh's model.

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