Numerical Investigation On The Effect Of Inclination Angle On Natural Convection Heat Transfer In A Rectangular Enclosure

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Abstract

The effect of inclination angle on convection heat transfer in a 2-D inclined rectangular enclosure is numerically investigation. Inclination angle varies from 30° to 60°. Governing equation were solved for Ra (10^3-10^4).

The enclosure object with heated left side wall, while the right side was cold, the top and bottom walls were adiabatic. The theoretical study involved the numerical solution of the Navier-Stokes and energy equations by explicit method using finite difference . The stream function – formulation was used in the mathematical model.

The numerical results showed that the main process of heat transfer is conduction for Rayleigh number less than $Ra \le 10^3$, and convection for Rayleigh number greater than $Ra > 10^3$. For $Ra=10^5$ and AR=3, show a bicellular flow in stream lines contour because the velocity nearby cold wall is larger than those which are far away from cold wall. In isotherm lines contour a plume region will be clearer. Average Nusselt number is increased with aspect ratio increasing for Ra ranging with (10^3-10^5) because of increasing of heat transfer area.

For a square enclosure filled with air (Pr=0.72)). There is an excellent agreement which validate the present computational model.

Keyword: Natural convection, heat transfer, rectangular enclosure, Navier-stokes equation.

التنبؤ العددي لتأثير انتقال الحرارة بالحمل على انتقال الحرارة في حيز مستطيل مغلق

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الخلاصة :

تأثير زاوية الميلان على انتقال الحرارة بالحمل في حيز مغلق مستطيل تم بحثه عدديا. وكانت زوايا الميل هي 30° و 60° . والمعادلات الحاكمة تم حلها لارقام رالي (10^3-10^4).

الحيز المستخدم هو ثنائي الأبعاد مع تسخين الجانب الأيسر من الحيز، في حين أن الجانب الأيمن كان باردا، وكانت الجدران العلوية والسفلية معزولة. الدراسة تضمنت حل معادلات نافير-ستوك والطاقة باستخدام طريقة الفروق المحددة. استخدمت صياغة الدوامية – دالة الانسياب في النموذج الرياضي.

وأظهرت النتائج العددية أن العملية الرئيسية لانتقال الحرارة هي بالتوصيل لرقم رالي أقل من 10^3 رأس، وانتقال الحرارة بالحمل لارقام رالي اكبر من 10^3. كما بينت النتائج ان دالة الانسياب تظهر بشكل خلية منفردة ما عدا حالة رقم رالي يساوي 10^5حيث تظهر بشكل ثنائي الخلايا عندما $AR=3$ بسبب ان السرعة قرب الجدار البارد هي اكبر من السرع البعيدة عن الجدار البارد. مخطط تساوي درجات الحرارة يبين منطقة الريشة بشكل واضح. معدل رقم نسلت يزداد بزيادة النسبة الباعية لارقام رالي (10^5-10^3) بسبب زيادة مساحة انتقال الحرارة.

لحيز مربع مملو بالهواء (لرقم براندتل=0.72)، التوافق بين النتائج ممتاز مما يؤكد صحة النموذج الحسابي الحالي.

Nomenclature

Symbols	**Meaning**
AR	Aspect ratio
C_p	Specific heat at constant pressure
E_{max}	Maximum error
G	Gravitational acceleration
H	Enclosure height
K	Thermal conductivity
W	Enclosure Width
Nu	Nussel number
$\overline{Nu}$	Average Nussel number
P	pressure
Pr	Prandtl number
Ra	Rayleih number
$r, r_\Psi, r_\Omega, r_\theta$	Relaxation parameter for stream function, vorticity, and temperature respectively
T	Temperature
U	Velocity component in x-direction
U	Dimensionless Velocity component in x-direction
V	Velocity component in y-direction
V	Dimensionless Velocity component in y-direction
x, y	Cartesian space coordinates
X,Y	Dimensionless Cartesian space coordinates

Greek Symbol

β	Coefficient of thermal expansion
ϕ	General dependent variable
μ	Molecular dynamic viscosity
ν	Kinematic viscosity
ω	Vorticity
Ω	Dimensionless vorticity
ρ	Density of fluid
ψ	stream function
Ψ	Dimensionless stream function
θ	Dimensionless temperature

Subscript Symbols

C, H	Related to cold and hot side respectively
(i,j)	Grid nodes in (x,y) direction

1. Introduction

Free or natural convection occurs when fluid motion is generated predominantly by body forces caused by density variations, under the earth's gravitational field. In the absence of the gravitational field, body forces may be caused by surface tension. The subject material here is focused on heat transfer with motion produced by buoyancy forces [1].

Convection in nature occurs in two different forms, the so-called natural convection and forced convection. Natural convection is the motion that results from the interaction of gravity with density differences within a fluid. The differences may result from gradients in temperature, concentration or composition [2,3].

In many engineering applications and naturally occurring processes, natural convection plays an important role as a dominating mechanism. Natural convection in enclosures has been widely used in many thermal applications such as in solar collectors, cooling devices for electronic instruments, building insulation, energy storage devices, furnace design and many others. Water is much used as a heat transfer fluid because of its availability and good thermal properties: it has good thermal conductivity, a large thermal capacity, low viscosity, and presents no hazard (usually just a spillage problem) [4].

This research presents a numerical study for natural convection in two-dimensional inclined enclosure, through the following research plan :-

i. Mathematical modeling of the problem including the governing equations (conservation and constitution laws) with the proper boundary conditions. After that these equations will be converted to dimensionless form then the stream function and vorticity (Ψ-Ω)

scheme will be applied on these equations and it will be solved numerically by finite difference technique.

ii. Make computational model to solve the derived mathematical model numerically. central finite Difference scheme will be used in the numerical formulation.

iii. A computer program will be built to perform the numerical calculation algorithm developed in (ii) above.

iv. Parametric study will be made by using the computer program to investigate the effect of various thermal parameters on the performance of the problem.

v. Verification of the developed computational algorithm through a comparison with previous available works and analysis and discussion of the results for final.

2. Mathematical Model

A schematic representation of the system under investigation is shown in **Figure (1)**, where L is side of the enclosure cavity which is calculated depending on the Rayleigh number. The gravity vector is directed in the negative *y* coordinate direction.

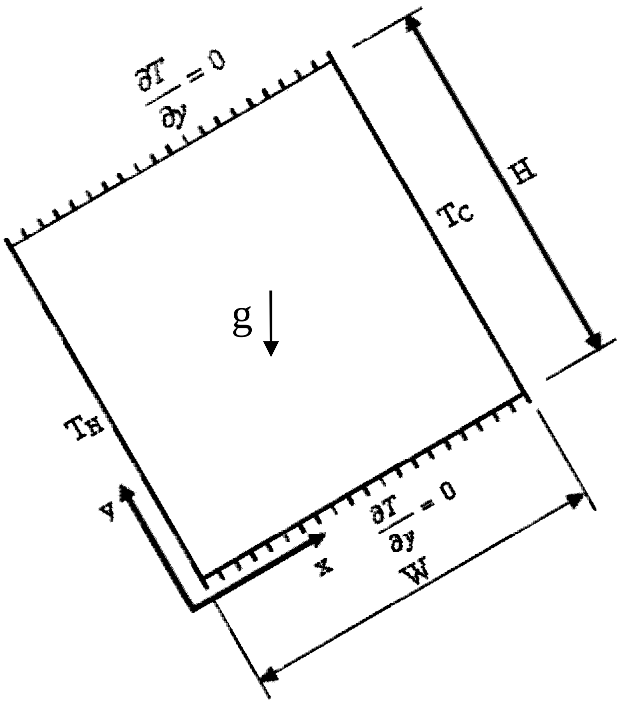

Fig. (1) : Schematic diagram of physical problem.

Continuity, momentum and energy equations for a two dimensional incompressible laminar flow has been solved using appropriate boundary conditions by mean computational fluid dynamics technique.

Following assumptions have been made: two-dimensional problem, there is no viscous dissipation, the gravity acts in the vertical direction, the fluid properties are constant and radiation heat exchange was assumed negligible. At steady state conditions using above assumption, the governing equations as are [5,6]:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

x-momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{r}\frac{\partial p}{\partial x}+n\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+gb\left(\mathrm{T}-\mathrm{T_C}\right)\cos f \tag{2}$$

y-momentum equation:

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{r}\frac{\partial p}{\partial x}+n\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+gb\left(T-T_C\right)\sin f \tag{3}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{n}{\Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The governing equations given above, i.e., eq. 1 to eq. 4, are given in terms of the so-called primitive variables, i.e., u, v, p, an T. the solution procedure discussed here is based on equations involving the stream function, the vorticity, and the temperature, as variables. The stream dimensionless function and vorticity are defined by:

$$u=\frac{\partial y}{\partial y},\ \mathrm{v}=-\frac{\partial y}{\partial \mathrm{x}} \tag{5}$$

$$w=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{6}$$

The vorticity equation is obtained by eliminating the pressure between the two momentum equations, i.e., by taking the y-derivative of **eq. 2** and subtracting from it the x-derivative of **eq. 3**. The equation of vorticity (conservation of angular momentum) becomes

$$\frac{\partial y}{\partial y}\frac{\partial w}{\partial x}-\frac{\partial y}{\partial x}\frac{\partial w}{\partial y}=n\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-bg\left(\frac{\partial T}{\partial x}\sin f-\frac{\partial T}{\partial y}\cos f\right) \tag{7}$$

In terms of stream function, the equation become

$$\left(\frac{\partial^2 y}{\partial x^2}+\frac{\partial^2 y}{\partial y^2}\right)=-w \tag{8}$$

While in terms of the stream function the energy equation becomes

$$\frac{\partial y}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial y}{\partial x}\frac{\partial T}{\partial y}=\frac{n}{\Pr}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{9}$$

Before considering the numerical solution to the above set of equations, it is convenient to rewrite the equations in terms of dimensionless variables. The following dimensionless variables will be used here:

$$\left.\begin{aligned}
&\Psi=\frac{y\ \Pr}{n},\ \Omega=\frac{w\mathrm{L}^2\ \Pr}{n}\\
&X=\frac{x}{W},\ \mathrm{Y}=\frac{\mathrm{y}}{\mathrm{H}}\\
&q=\frac{T-T_c}{T_h-T_c}\\
&Ra=\frac{gb\,(T_H-T_C)L^3}{an}=\frac{gb\,(T_H-T_C)L^3}{n^2}\Pr
\end{aligned}\right\} \tag{10}$$

In terms of these variables, the stream function, vorticity and energy equations respectively becomes

$$\left(\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}\right)=-\Omega \tag{11}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\Omega}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\Omega}{\partial Y}=\Pr\left(\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right)-Ra\frac{\partial q}{\partial X} \tag{12}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial q}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial q}{\partial Y}=\left(\frac{\partial^2 q}{\partial X^2}+\frac{\partial^2 q}{\partial Y^2}\right) \tag{13}$$

2.1 Boundary Conditions

The boundary conditions of velocity and temperature fields are shown in **Figure. (1)** presented as [7]:

$$\left.\begin{array}{l} X=0:\mathrm{u}=\mathrm{v}=\Psi=0,q=1,\text{-}\Omega=\dfrac{\partial^2\Psi}{\partial X^2} \\ \mathrm{X}=1:\ \mathrm{u}=\mathrm{v}=\Psi=0,q=0,\text{-}\Omega=\dfrac{\partial^2\Psi}{\partial X^2} \\ \mathrm{Y}=0:\mathrm{u}=\mathrm{v}=\Psi=0,\dfrac{\partial q}{\partial Y}=0,\text{-}\Omega=\dfrac{\partial^2\Psi}{\partial Y^2} \\ \mathrm{Y}=1:\mathrm{Y}=0:\mathrm{u}=\mathrm{v}=\Psi=0,\dfrac{\partial q}{\partial Y}=0,\text{-}\Omega=\dfrac{\partial^2\Psi}{\partial Y^2} \end{array}\right\} \qquad (14)\mathbf{3.}$$

3. Numerical Solution

The method of numerical solution used is the central finite difference scheme technique to convert the partial differential equation to an algebraic which can be solved numerically. The energy equation is a nonlinear partial differential equation, which has (convection terms) on the left hand side of the **equ. 13** and (diffusion term) on the right hand side[8,9,11].

3.1 Nusselt Number Calculation

The local heat transfer rate along the heated wall is obtained from the heat balance that gives an expression for local Nusselt number as

$$Nu=-\left.\frac{\partial q}{\partial X}\right|_{\mathrm{X}=0} \quad ; \quad 0\le Y\le 1 \qquad (15)$$

The average value for Nusselt number is defined by

$$\overline{Nu}=\frac{1}{H}\int_0^H NudY \qquad (16)$$

This equation can be readily evaluated using Simpson's rule.

4. Results and discussion

A computational algorithm for laminar natural convection in two-dimensional inclined rectangular enclosure is used. All the results are calculated for air as working fluid (Pr=0.72), while Ra numbers ranging within ($10^3 \le Ra \le 10^5$). Finer uniform grid (41×41) for Ra=10^3, (51×51) for Ra=10^4 and (61×61) for Ra=10^5 at aspect ratio (AR=1). The iterative routine, the selected grid and the relaxation parameters are verified at different Rayleigh numbers and inclination angles (ϕ=30°, 60°) respectively.

Figure (2) show the effect of inclined angles for Ra=103 and ϕ=30° and 60° at AR=1.0, 2.0 and 3.0 where, the fluid near the hot wall is heated causing the density to decrease and the fluid will start to move nearby the hot wall towards the cold wall. From the stream function plots it can be seen that all three configurations contain a single rotating vortex which takes the shape of the cavity. By looking at the sign of the gradient for the stream function in the x-direction, we can find that the vortices are rotating in the clockwise direction. The vortex moves the warm fluid from the left wall along the top of the geometry and results in higher temperatures at the top half of cavity and symmetric behavior at Ra=10^3 and ϕ=60°.

Figure (3) show the effect of inclined angles for Ra=104 and ϕ=30° and 60° at AR=1.0, 2.0 and 3.0 on mechanism of similar behavior is noticed.

Figure (4) show the effect of inclination angle on stream function contour for Ra=10^5. This effect is more pronounced at high Rayleigh numbers where multicellular flow patterns dominate. The strength of the vortices is also increased as seen by the magnitude of the stream function gradient.

Figures (5, 6 and 7) represent the effect of inclination angle on isotherm contour for Ra=10^3, 10^4 and 10^5. From these figures the heat transfer operation can be divided into multi region, first region is that of heat transfer by conduction which extended to (Ra=10^3). The second region is transition region until plum region is appeared and ranged ($10^4 \leq$Ra $\leq 10^5$). Third region is the plume region which is at (Ra$\geq 10^5$), so when Ra increase plume region will be more clear especially at (Ra=10^5). The parallel isothermal lines away from the heated wall at low Rayleigh number, Ra=10^2 and 10^3, indicate conduction dominated mode of heat transfer, though convection is developed in the layer adjacent to the heated wall.

Figures (8, 9) show the variation of average Nusselt number along vertical hot wall for (Ra=10^3, 10^4,10^5) and inclined angle (30°and 60°). They show the temperature increases with increasing Ra. Nusselt is number increasing near the hot wall at inclined angle 30° and Ra=10^3, 10^4, and decrease average Nusselt number at inclined angle 60° for all aspect ratios and Ra numbers.In order to validate the numerical model, the results of variation local Nusselt number and a relation between average Nusselt number and Ra number, are compared with previous works. For square enclosure filled by air (Pr=0.72) the results are compared with [10]. There are agreements in results and excellent agreement which validate the present computational model as shown **in Figure (10).**

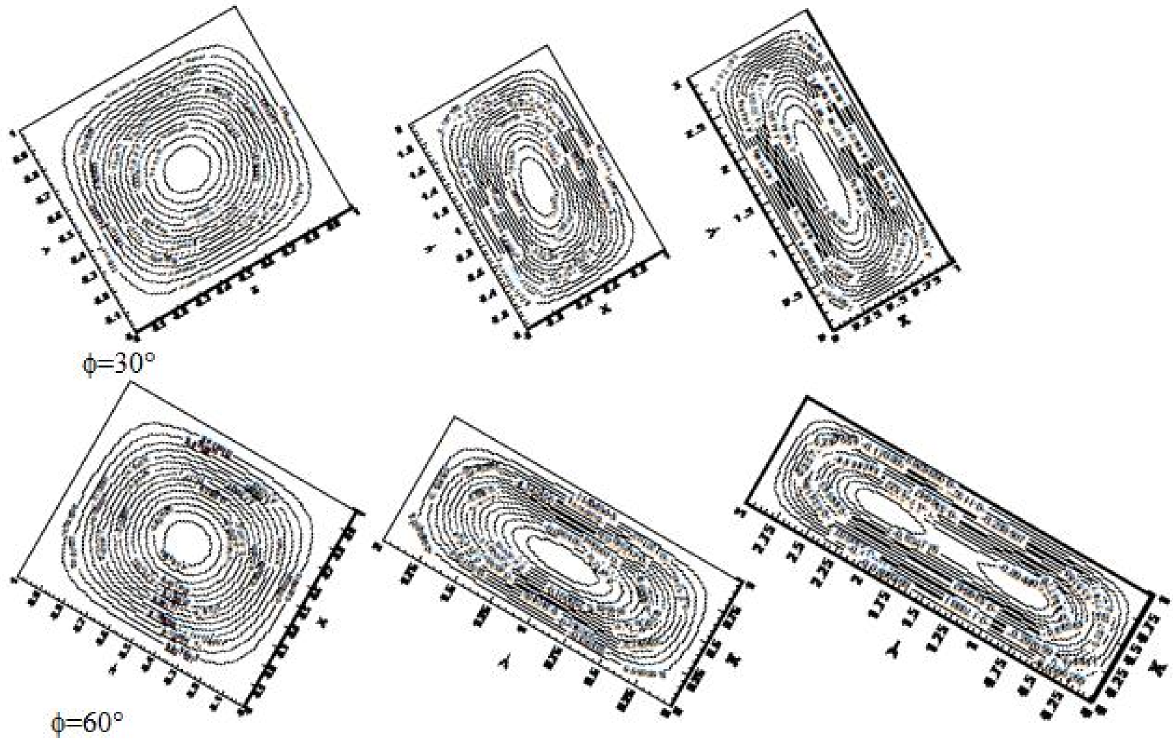

Fig. (2): effect of inclination angle on stream function contour for Ra=10^3.

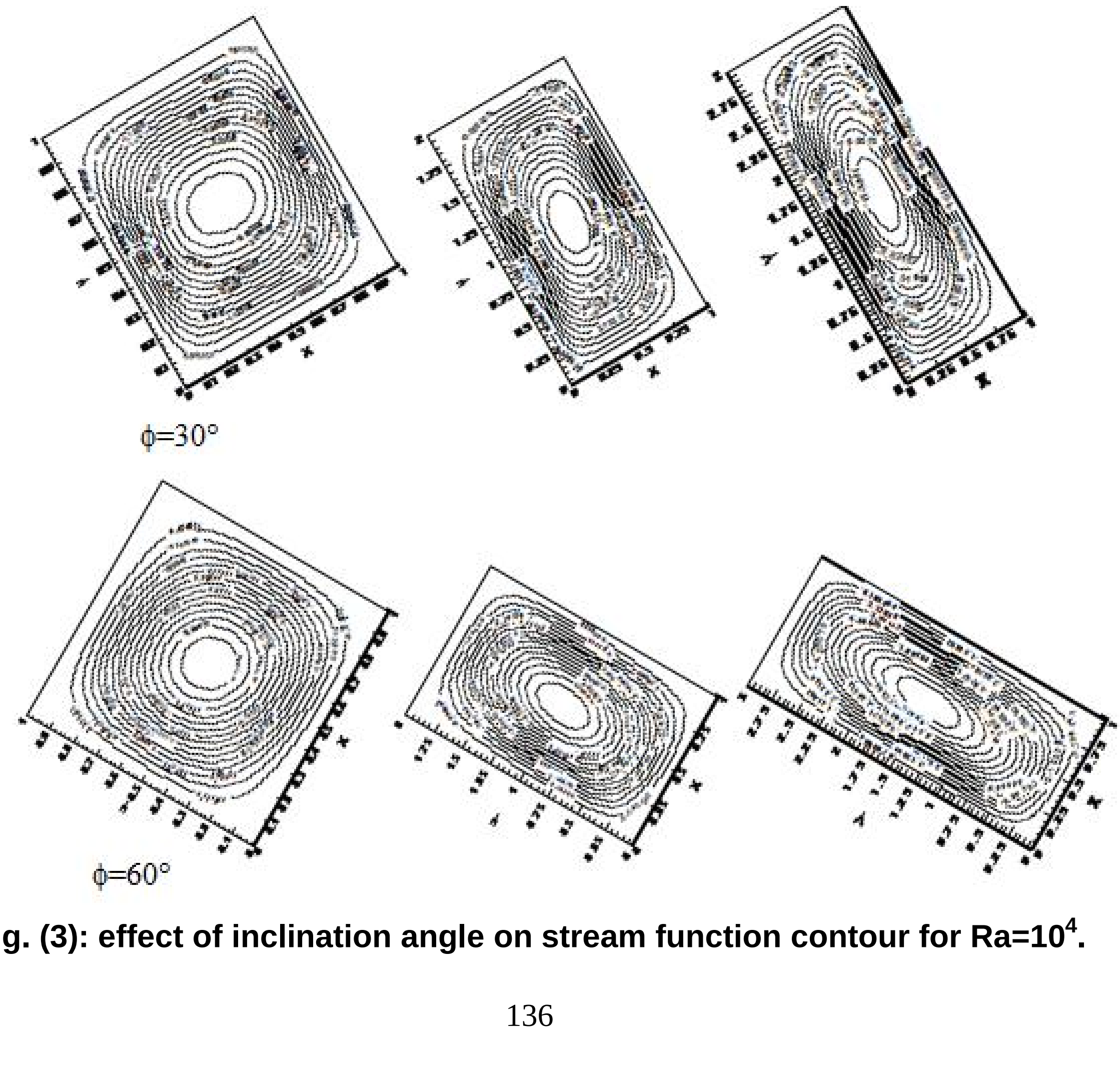

Fig. (3): effect of inclination angle on stream function contour for Ra=10^4.

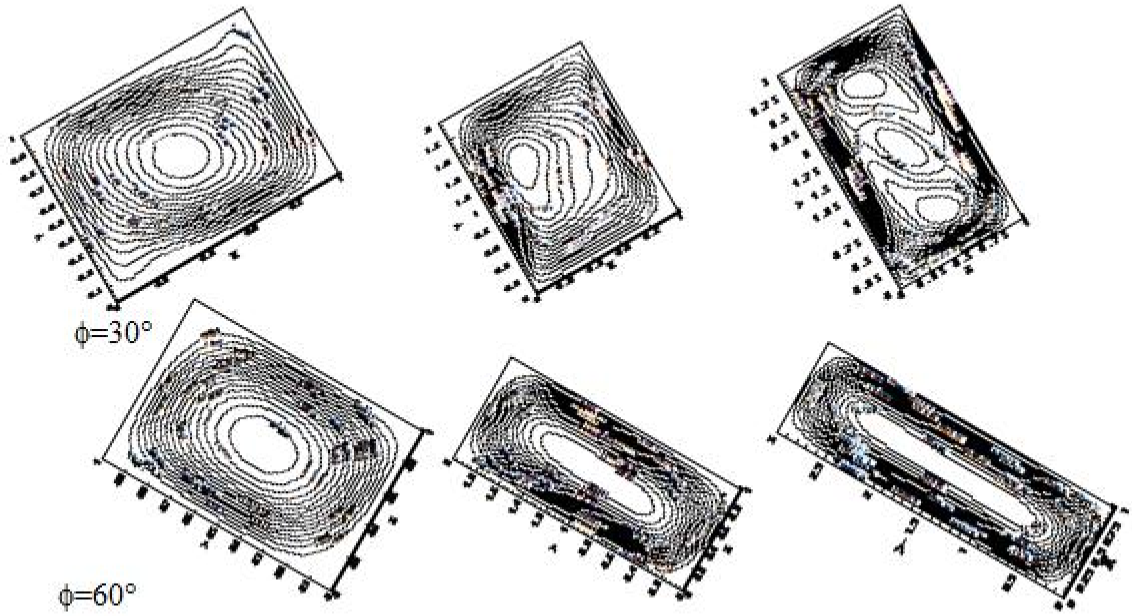

Fig. (4): effect of inclination angle on stream function contour for

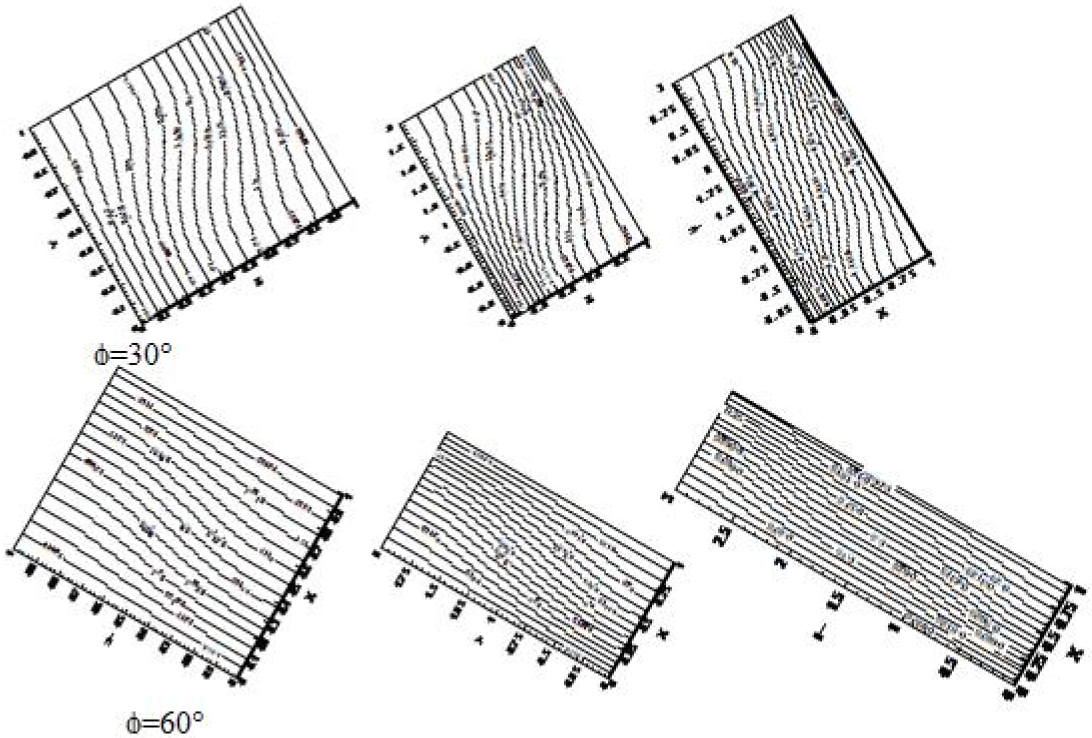

Fig. (5): effect of inclination angle on isotherm contour for Ra=10^3.

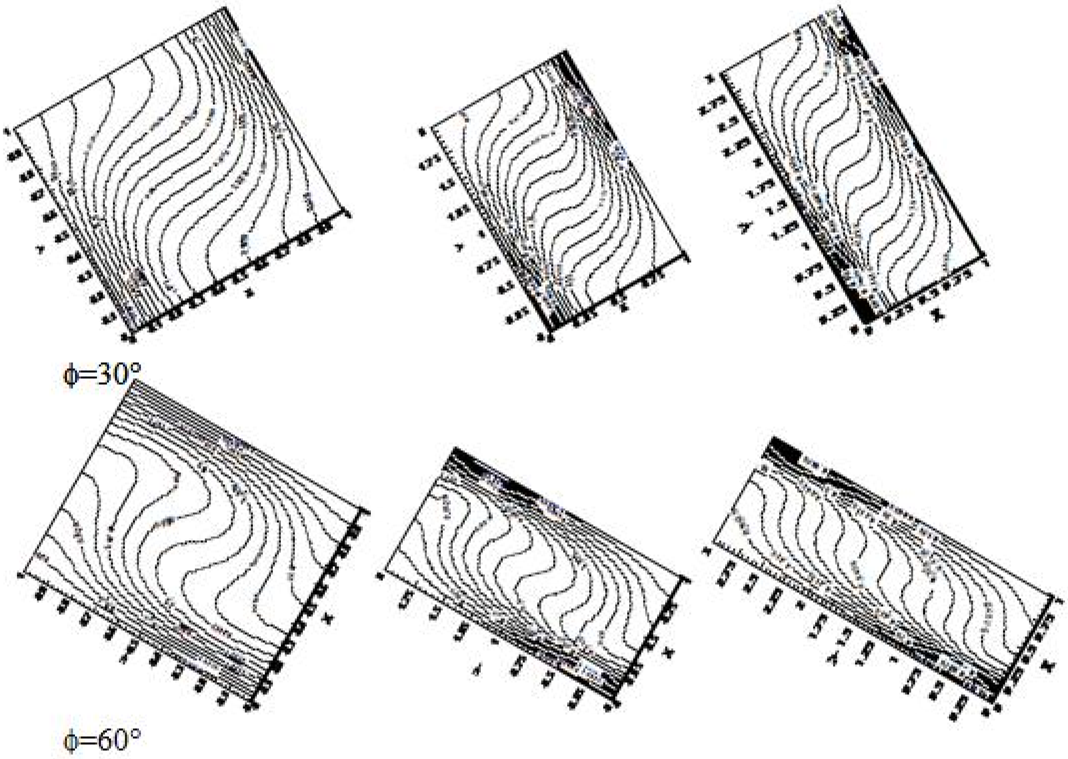

Fig. (6): effect of inclination angle on isotherm contour for Ra=10^4.

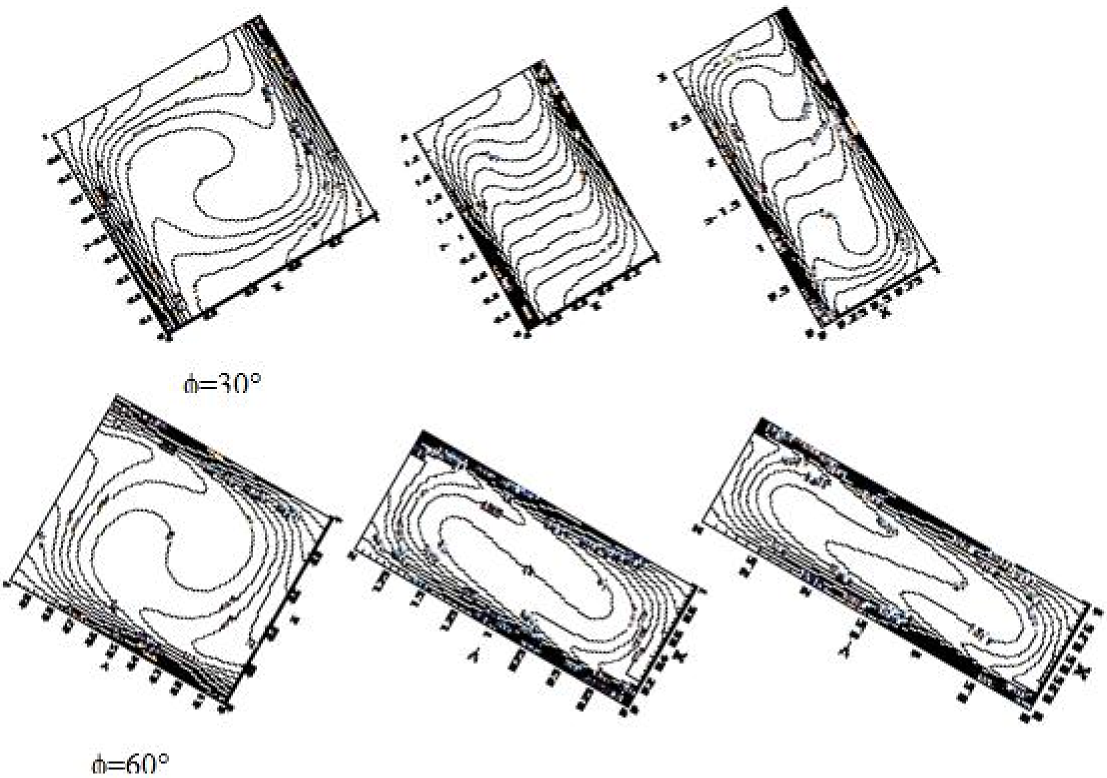

Fig. (7): effect of inclination angle on isotherm contour for Ra=10^5.

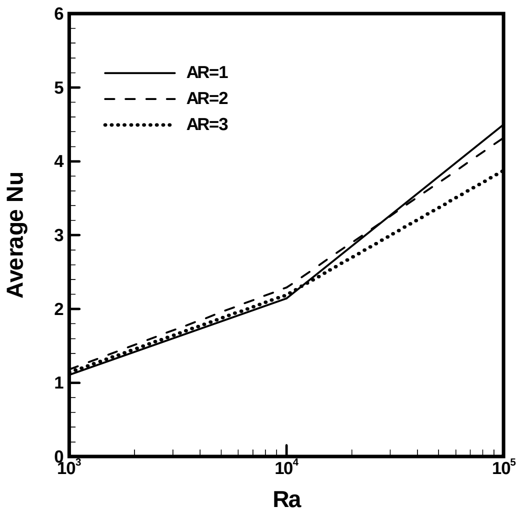

Fig.(8): effect AR on average number for different Ra No. at f=30°.

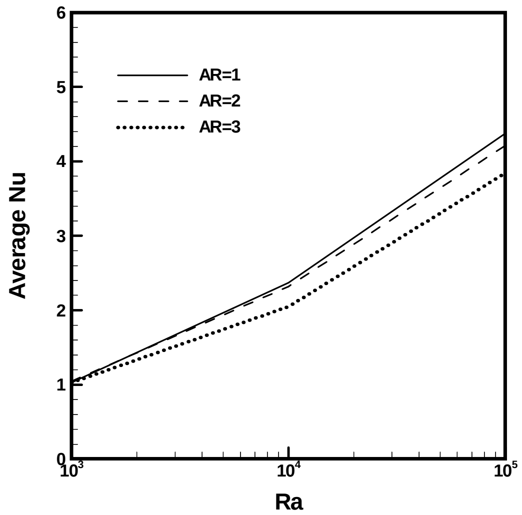

Fig.(9): effect AR on average number for different Ra No. at f=60°.

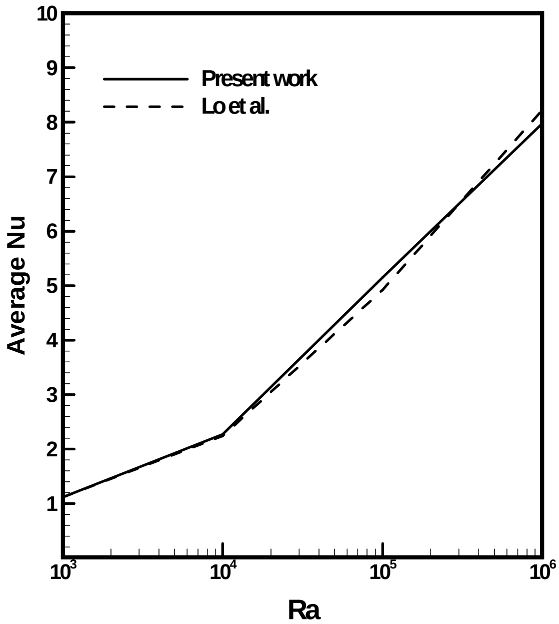

Fig.(10): show comparison between present work and Lo et al. for relation between

5. Conclusions

In this research natural convection in an inclined enclosure is studies. The enclosure consists of top and bottom adiabatic surfaces, two isotherm vertical walls (left is hot and right is cold). The partial differential equations for two dimensional in stream-vorticity form and energy equation are solved based on central finite difference scheme. The solution scheme is validated by comparison with last research, where the agreement was excellent. For ($Ra=10^3$) the heat transfer operation by conduction, and convection for ($Ra>10^3$). The streamlines show a single cell form a bicellular form. Nusselt number increasing near the hot wall at inclined angle 30° and $Ra=10^3$, 10^4, and decrease average Nusselt number at inclined angle 60° for all aspect ratio and Ra numbers.

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